Worksheet - August 20   MAT 3501   Fall 2018

1. (a) Use a direct proof to show that the sum of three consecutive integers is a multiple of 3.

(b) Is the sum of three consecutive integers always a multiple of 6?

2. (a) Prove that the sum, difference, product, or quotient of two rational numbers is also rational. For quotient, assume additionally that the denominator is not zero.

(b) Prove that the sum of a rational number with an irrational number is irrational. (Hint: Use contradiction and part (a).)

(c) Formulate (carefully!) a similar statement as (b) for product.

(d) Is the sum or product of two irrational numbers necessarily irrational?

3. The literature teacher decided to find out who out of 40 students had read the books A, B, C over the summer break. The results were the following: 25 students had read the book A, 22 the book B and 22 the book C; 33 students had read the book A or B, 32 the book A or C, and 31 had read the book B or C; 10 students had read all the three books.

(a) How many students had read exactly one book (of the three)?

(b) How many students had read none of the books?

4. Suppose n lines are drawn in the plane so that no two lines are parallel and no three lines are concurrent. Find a formula, in terms of n, for the number of regions determined in the plane.

Hint: The obvious hint for such problems is to start with small values of n and investigate for an eventual pattern.