Name: ____________________________ **Panther ID:** ______________

Worksheet - Sep. 21 MAT 3501 Fall 2017

1. (a) Use the fundamental theorem of algebra to show that any polynomial with real coefficients can be decomposed as a product of linear or quadratic factors, **all with real coefficients**.

(b) You mention the above result to your students and one comes up and says: "I thought that a polynomial like $x^4 + 1$ cannot be factored." How do you answer?

(c) Find a factoring as in part (a) for the polynomial $p(x) = x^3 + 2$.

2. Suppose that $p(x)$ is a polynomial of odd degree with real coefficients. Show that the equation $p(x) = 0$ must have at least one real real solution.

3. Let $x_1, x_2, x_3 \in \mathbf{C}$ be the roots of the polynomial $p(x) = x^3 + 2x^2 - 3x + 4$. Use Viete's relations to find:

(a) $x_1x_2x_3$

(b) $x_1^2 + x_2^2 + x_3^2$

(c) $x_1^3 + x_2^3 + x_3^3$

4. Let $x_1, x_2, \ldots, x_n \in \mathbf{C}$ be the roots of the polynomial $p(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$.

(a) Find the roots of the polynomial $\widetilde{p}(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$.

(b) Find $\frac{1}{x_1^2} + \frac{1}{x_2^2} + \ldots + \frac{1}{x_n^2}$.

5. Determine all polynomials $p(x)$ so that

$p(x^2+1) = (p(x))^2+1$ and $p(0) = 0$.