Name: Solution Key PantherID: ____________

Quiz 1 - Topology – Fall 2015

1. (9 pts) Define each of the following notions:

(a) connected topological space;

A space which cannot be written as the disjoint union of two non-empty open subsets.

(b) path connected topological space (you can assume known the definition of a path);

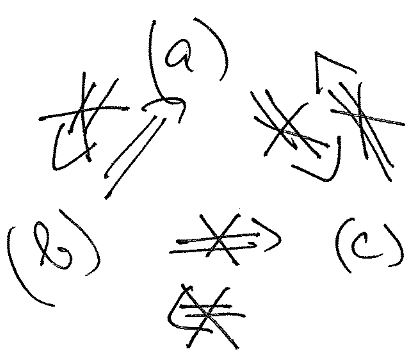
A space in which every two points can be joined by a path. ($\forall\, a,b \in X$, $\exists\, f:[0,1] \to X$ continuous, $f(0)=a$, $f(1)=b$.)

(c) locally path connected topological space;

A space X so that for every $p \in X$ and for every nbd. U of x, there exists a nbd V of x so that $V \subseteq U$ and V is path connected.

2. (4 pts) Make a diagram (or a list) resuming implications among the notions in Problem 1.

$(b) \Rightarrow (a)$ and $((a) \wedge (c)) \Rightarrow (b)$ are true but other implications (as in the diagram) are not.



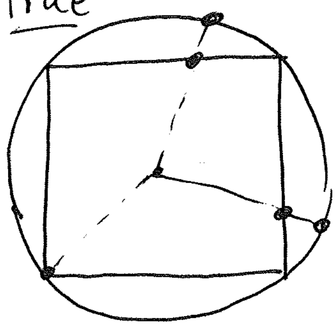
You should think at the counter-examples (although they were not required)

E.g. - Topologist's comb shows that $(a) \not\Rightarrow (c)$ and $(b) \not\Rightarrow (c)$

3. (12 pts) True or False? Answer (2 pts) and give a brief justification (2 pts) in each case.

(a) A square is homeomorphic to a circle.

True



One way to describe a homeomorphism is to inscribe the square in the circle and use a radial map from the center

(b) Letters " T " and " X " are homeomorphic.

False : 

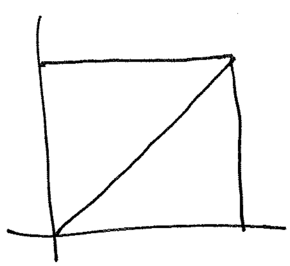
"X" has a cut point p so that $X \setminus \{p\}$ has 4 connected components, whereas for any cut point q of T, $T \setminus \{q\}$ has at most 3 connected components

(c) There exists a non-connected set $A \subseteq \mathbb{R}^2$, so that both $p_1(A)$ and $p_2(A)$ are connected sets in $\mathbb{R}$. Here $p_1, p_2 : \mathbb{R}^2 \to \mathbb{R}$ denote the projections on the first, respectively, second components $p_1((x, y)) = x$, $p_2((x, y)) = y$.

True : Let, for instance,

$$A = [0,1] \times [0,1] \setminus \Delta$$

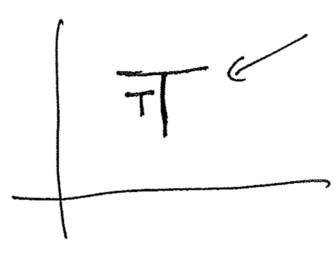
where $\Delta = \{(x,x) \mid x \in [0,1]\}$.



A is disconnected

but $p_1(A) = [0,1]$ and $p_2(A) = [0,1]$ are both connected

Many other examples possible, of course



A is the union of those "T"'s