Pb. 2, p. 126. Show that $\mathbf{R} \times \mathbf{R}$ with the dictionary order topology is metrizable.

Sketch of the solution. You fill in the details.

1. Observe that the dictionary order topology on $\mathbf{R} \times \mathbf{R}$ is the same as the product topology $\mathbf{R}_d \times \mathbf{R}$ of the discrete topology on the first factor, and the the standard topology on the second (Pb. 9, sect. 16). For this, observe that the dictionary order topology on $\mathbf{R} \times \mathbf{R}$ can be generated just by vertical open segments (the stripes can be written as union of such segments).

2. You know that $\mathbf{R}_d$ is metrizable, using the discrete metric; also, the standard topology on $\mathbf{R}$ is given by the standard metric. But the product of two metrizable spaces is metrizable (this is true finite products and even for infinite *countable* products - Ex. 3, Sect. 21).

If d_X is a metric on X and d_Y is a metric on Y, then

$$d((x, y), (x', y')) = max(d_X(x, x'), d_Y(y, y'))$$

is a metric on $X \times Y$ (check this).

Moreover, the topology induced by d is the same as the product topology (check this also).