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**Homework 1 - Topology**

**Due Wednesday, Jan. 23, 2008**

1. (a) Let  $X$  be a set with more than one element. Show that there exists a metric  $d$  on  $X$ , so that the topology generated by  $d$  is the discrete topology on  $X$ .  
(b) Let  $X$  be a set with more than one element. Show that for no metric  $d$  on  $X$ , the topology generated by  $d$  is the same as the trivial topology on  $X$ .  
(c) Let  $X = \{a, b, c\}$ . Give an example of a topology on  $X$ , other than the trivial topology, that cannot be the topology generated by a metric on  $X$ .
  
2. (a) Consider  $\mathbf{R}_l$ , that is,  $\mathbf{R}$  endowed with the lower limit topology (see notes or textbook). Show that a sequence  $\{x_n\}_n$  converges to  $x$  in the lower limit topology if and only if  $\{x_n\}_n$  converges to  $x$  in the standard topology and  $x_n \geq x$  for all except finitely many values of  $n$ .  
(b) Consider  $\mathbf{R}_K$ , that is,  $\mathbf{R}$  endowed with the topology  $\mathcal{T}_K$  (see notes or textbook). Formulate and prove the equivalent condition for the convergence of a sequence in  $\mathbf{R}_K$ .