Solution Homework 3: MAA 4211 Spring 2002

4, page 37. (a) $|x_n - a| \leq b_n$ for large n means that there exists $N_0 \in \mathbf{N}$ such that

$$|x_n - a| \leq b_n, \ \forall n \geq N_0.$$

Let $\epsilon > 0$. Since $b_n \to 0$, there exists $N_1 \in \mathbf{N}$, $N_1 \geq N_0$, such that $|b_n| < \epsilon, \ \forall n \geq N_1$.
Thus, for any $n \geq N_1$, we have

$$|x_n - a| \leq b_n < \epsilon.$$

(b) The conclusion that x_n converges to a remains true. The proof is as above, except N_1 is chosen now so that $N_1 \geq N_0$ and $|b_n| < \epsilon/C, \ \forall n \geq N_1$.
□

8, page 37. ($\Rightarrow$) Follows directly from Theorem 3.6.

($\Leftarrow$) We assume that *any* subsequence $\{x_{n_k}\}_k$ of $\{x_n\}_n$ converges to a and want to prove that $\{x_n\}_n$ itself must converge to a.
We consider a particular subsequence by taking $n_k = k$ (notice that $n_k < n_{k+1}$). By assumption, the subsequence $\{x_{n_k}\}_k$ converges to a, but with our special choice $x_{n_k} = x_k$, so the subsequence is nothing but the sequence itself.
□