Solution Homework 6: MAA 4211 Spring 2002

2 (b), (c), page 78. (b) The functions $g(x) = 1 - x$, $h(x) = 1 + x$ are continuous on $[0, 1]$ and $h(x) \neq 0$ for any $x \in [0, 1]$. By Theorem 3.22, it then follows that $f(x) = g(x)/h(x)$ is continuous on $[0, 1]$. □

(c) As in part (b), using Theorems 3.22 and 3.24, the function $f(x)$ is continuous at any $x \neq 0$. The only issue is the continuity at 0. But since the function $\sin(1/x)$ is bounded and $\sqrt{x} \to 0$ as $x \to 0_+$, from the squeeze theorem for functions (Theorem 3.9 (ii)) it follows that $\lim_{x \to 0_+} \sqrt{x} \sin(1/x) = 0$. □

4, page 78. The condition $f(a) < M$ is equivalent to $M - f(a) > 0$. Because $f(x)$ is continuous at a, there exists $\delta > 0$ such that

$$-(M - f(a)) < f(x) - f(a) < M - f(a), \quad \forall x \in (a - \delta, a + \delta).$$

The right side of this inequality implies that $f(x) < M$, $\forall x \in (a - \delta, a + \delta)$. □