To receive credit you MUST SHOW ALL YOUR WORK.

1. (20+5 pts) For all parts of the problem let $A = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$ and do the change of variables

$$y_1 = 3x_1 - 2x_2, \quad y_2 = x_1 + x_2.$$

(a) Solve the equation $\dfrac{d\mathbf{x}}{dt} = A\mathbf{x}$, where $\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$.

That is, express $x_1(t)$ and $x_2(t)$ in terms of the initial conditions $x_1(0), x_2(0)$.

(b) Solve the equation $\dfrac{d^2\mathbf{x}}{dt^2} = -A\mathbf{x}$.

That is, express $x_1(t)$ and $x_2(t)$ in terms of the initial conditions (values and first derivatives at $t = 0$).

(c) (5 bonus points) Explain how the change of variables was found and why it works to decouple linear systems involving the given matrix A.