Midterm Exam - MAS 5145 Summer06 **NAME:** ____________________

To receive credit you MUST SHOW ALL YOUR WORK.

1. (12 pts) Define each of the following (4 pts each):

(a) Span$\{\mathbf{v}_1, ..., \mathbf{v}_k\}$, where $\mathbf{v}_1, ..., \mathbf{v}_k$ are vectors in a vector space V.

(b) Basis of a vector space V

(c) The eigenspace E_λ corresponding to an eigenvalue λ of a $n \times n$ matrix A

2. (14 pts) Show that a linear transformation is one to one (injective) if and only if its kernel is $\{\mathbf{0}\}$.

3. (12 pts) Suppose $L : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, that $L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. What is $L\begin{pmatrix} 3 \\ 2 \end{pmatrix}$? In general, what is $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$?

4. (14 pts) Let $A = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$. Compute A^n.

5. (16 pts) In $\mathbb{R}_2[t]$, let $\mathbf{b}_1 = 1$, $\mathbf{b}_2 = 1 + t$, $\mathbf{b}_3 = t + t^2$.

(a) (8 pts) Show that $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis in $\mathbb{R}_2[t]$.

(b) (8 pts) Let $d/dt : \mathbb{R}_2[t] \to \mathbb{R}_2[t]$ be the derivative map. Find the matrix of d/dt with respect to the the basis $\mathcal{B}$ from part (a).

6. (22 pts) Let $a, b \in \mathbb{R}$, with $b \neq 0$.

(a) (8 pts) Show that the matrix $A = \begin{pmatrix} a & 0 \\ b & a \end{pmatrix}$ is not diagonalizable.

(b) (14 pts) Find the solution $\mathbf{x}(t)$ of the coupled equations $d\mathbf{x}/dt = A\mathbf{x}$, with $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

7. (10 pts)

Is the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ diagonalizable? Justify your answer.

8. (10 pts) Show that two conjugate matrices have the same eigenvalues.