Name: Solution Key Panther ID: ____________

Worksheet week 11 Calculus I Spring 2015

1. Give a complete graph for $f(x) = \frac{\ln x}{x^2}$. The actual graph should be supported by your work, which should include: domain, critical points (coordinates and type of critical points), sign chart for f' and f'', inflection points, end behavior of the function and eventual asymptotes (vertical/horizontal).

Domain: $x \in (0, +\infty)$

$$f'(x) = \left(\frac{\ln x}{x^2}\right)' = \frac{\frac{1}{x}\cdot x^2 - (\ln x)\cdot 2x}{x^4} = \frac{x(1-2\ln x)}{x^4} = \boxed{\frac{1-2\ln x}{x^3}}$$

Critical point(s): $f'(x)=0 \iff 1-2\ln x = 0 \iff \ln x = \frac{1}{2} \Rightarrow \boxed{x = e^{\frac{1}{2}}}$

~~Sign chart~~

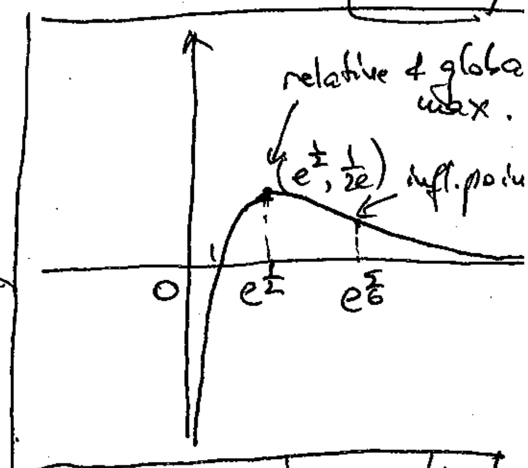
$$f''(x) = \left(\frac{1-2\ln x}{x^3}\right)' = \frac{-2\cdot\frac{1}{x}\cdot x^3 - (1-2\ln x)\cdot 3x^2}{x^6} = \frac{-2x^2-3x^2+6x^2\ln x}{x^6}$$

$$\boxed{f''(x) = \frac{-5+6\ln x}{x^4}}$$ so $f''(x)=0 \iff -5+6\ln x = 0 \Rightarrow \boxed{x = e^{\frac{5}{6}}}$

Sign chart

relative maximum. inflection point

x	0	$e^{\frac{1}{2}}$	$e^{\frac{5}{6}}$	$+\infty$
$f'(x)$	$+ + + + +$	0	$- - - - -$	$-$
$f(x)$	← concave down →		← concave up	
$f''(x)$	$- - - - - - -$		0	$+ + + +$



End behavior / possible asymptotes for $f(x)$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\ln x}{x^2} = \frac{-\infty}{0^+} = -\infty$$ so $x=0$ is a vertical asymptote

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\frac{\ln x}{x^2} \overset{\frac{\infty}{\infty}}{\underset{l'H}{=}} \lim_{x\to+\infty}\frac{\frac{1}{x}}{2x} = \lim_{x\to+\infty}\frac{1}{2x^2} = \boxed{0}$$ so $y=0$ is a horiz asymptote

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y ~~Coord.~~ of ~~~~ critical point $\quad y = f(e^{\frac{1}{2}}) = \frac{\ln(e^{\frac{1}{2}})}{(e^{\frac{1}{2}})^2} = \frac{\frac{1}{2}}{e} = \frac{1}{2e}$

Coord. of x-axis intercept

$y = \frac{\ln x}{x^2} = 0 \Rightarrow$ ~~~~ $x = 1$