

NAME: Solution Key

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Worksheet week 3 - MAC 2311, Spring 2015

1. Compute:

$$(a) \lim_{x \rightarrow 0} \frac{\tan^2(3x)}{x \sin(5x)} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{\tan^2(3x) \cdot 9x^2}{x \cdot \frac{\sin(5x)}{5x} \cdot 5x}$$

$$= \frac{9}{5}$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin(3x^2) + x^2}{\sin^2(3x)} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{\frac{\sin(3x^2)}{3x^2} \cdot 3x^2 + x^2}{\left(\frac{\sin(3x)}{3x}\right)^2 \cdot 9x^2}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 \left[ \frac{\sin(3x^2)}{3x^2} \cdot 3 + 1 \right]}{\frac{\sin^2(3x)}{(3x)^2} \cdot 9x^2} = \frac{4}{9}$$

$$(c) \lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{x^2} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{(1 - \cos(3x))(1 + \cos(3x))}{x^2(1 + \cos(3x))}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2(3x)}{x^2(1 + \cos(3x))} = \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{x^2(1 + \cos(3x))}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin^2(3x)}{(3x)^2} \cdot 9x^2}{x^2(1 + \cos(3x))} = \frac{9}{2}$$

$$(d) \lim_{x \rightarrow +\infty} \frac{1 - \cos(3x)}{x^2} \quad \text{something bounded but oscillates}$$

Apply Squeeze Theorem

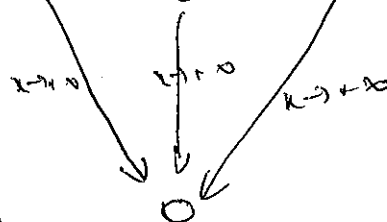
$$-1 \leq \cos(3x) \leq 1 \quad | \cdot (-1)$$

$$1 \geq -\cos(3x) \geq -1 \quad | \text{Add } 1$$

$$2 \geq 1 - \cos(3x) \geq 0$$

$$\text{or } 0 \leq 1 - \cos(3x) \leq 2 \quad (\text{Divide by } x^2 \text{ as } x^2 \rightarrow \infty)$$

$$0 \leq \frac{1 - \cos(3x)}{x^2} \leq \frac{2}{x^2}$$



$$\text{Thus } \lim_{x \rightarrow +\infty} \frac{1 - \cos(3x)}{x^2} = 0$$

$$(e) \lim_{x \rightarrow +\infty} x \tan(3/x) = \infty \cdot 0$$

$$= \lim_{x \rightarrow +\infty} \frac{\tan(3/x)}{1/x} =$$

$$= \lim_{x \rightarrow +\infty} \frac{3 \cdot \tan(3/x)}{3/x} = \text{Subst. } y = \frac{3}{x} \text{ when } x \rightarrow +\infty \text{ } y = \frac{3}{x} \rightarrow 0^+$$

$$= \lim_{y \rightarrow 0^+} \frac{3 \cdot \tan(y)}{y} = 3$$