NAME: Solution Key Panther ID: _____

Worksheet week 3 - MAC 2311, Spring 2015

1. Compute:

(a) $\lim_{x\to 0} \frac{\tan^2(3x)}{x\sin(5x)} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\frac{\tan^2(3x)}{(3x)^2}\cdot 9x^2}{x\cdot\frac{\sin(5x)}{5x}\cdot 5x}$

$= \frac{9}{5}$

(b) $\lim_{x\to 0} \frac{\sin(3x^2)+x^2}{\sin^2(3x)} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\frac{\sin(3x^2)}{3x^2}\cdot 3x^2 + x^2}{\frac{\sin^2(3x)}{(3x)^2}\cdot 9x^2} =$

$= \lim_{x\to 0} \frac{x^2\left[\frac{\sin(3x^2)}{3x^2}\cdot 3 + 1\right]}{\frac{\sin^2(3x)}{(3x)^2}\cdot 9x^2} = \frac{4}{9}$

(c) $\lim_{x\to 0} \frac{1-\cos(3x)}{x^2} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{(1-\cos(3x))(1+\cos(3x))}{x^2(1+\cos(3x))}$

$= \lim_{x\to 0} \frac{1-\cos^2(3x)}{x^2(1+\cos(3x))} = \lim_{x\to 0} \frac{\sin^2(3x)}{x^2(1+\cos(3x))}$

$= \lim_{x\to 0} \frac{\frac{\sin^2(3x)}{(3x)^2}\cdot 9x^2}{x^2(1+\cos(3x))} = \frac{9}{2}$

(d) $\lim_{x\to+\infty} \frac{1-\cos(3x)}{x^2}$ something bounded but not oscillating / ∞

Apply Squeeze Theorem

$-1 \le \cos(3x) \le 1 \quad | \cdot(-1)$

$1 \ge -\cos(3x) \ge -1 \quad |$ Add 1

$2 \ge 1-\cos(3x) \ge 0$

or $0 \le 1-\cos(3x) \le 2 \quad |$ Divide by x^2 $(x^2>0)$

$0 \le \frac{1-\cos(3x)}{x^2} \le \frac{2}{x^2}$

$x\to+\infty$ $\quad$ $x\to+\infty$ $\quad$ $x\to+\infty$

0

Thus $\lim_{x\to+\infty} \frac{1-\cos(3x)}{x^2} = 0$

(e) $\lim_{x\to+\infty} x\tan(3/x) = \infty\cdot 0$

$= \lim_{x\to+\infty} \frac{\tan\left(\frac{3}{x}\right)}{\frac{1}{x}} =$

$= \lim_{x\to+\infty} \frac{3\cdot\tan\left(\frac{3}{x}\right)}{\frac{3}{x}} =$ subst. $y = \frac{3}{x}$ when $x\to+\infty$ $y = \frac{3}{x} \to 0^+$

$= \lim_{y\to 0^+} 3\cdot\frac{\tan(y)}{y} = 3$