Integration by parts formula:

$$\int (du)v = uv - \int u(dv)$$

Proof: Given differentiable functions $u(x), v(x)$, the product rule gives

$$\Big(u(x)v(x)\Big)' = u'(x)v(x) + u(x)v'(x).$$

We integrate both sides:

$$\int \Big(u(x)v(x)\Big)' \, dx = \int \Big(u'(x)v(x) + u(x)v'(x)\Big) \, dx \; .$$

The left term is just $u(x)v(x)$. Splitting the right term in a sum of two integrals, we get

$$u(x)v(x) = \int u'(x)v(x) \, dx + \int u(x)v'(x) \, dx \; .$$

As $du = u'(x)dx$ and $dv = v'(x)dx$, and omitting the dependence of x, the above relation can be written as

$$uv = \int (du)v + \int u(dv)$$

This is just the integration of parts formula, up to rearranging terms. QED