**Name:** ____________________ **Panther ID:** ____________

**Exam 3** Calculus II Fall 2019

**To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.**

**1.** (12 pts) The first six terms of a sequence $\{a_n\}_n$ are given below

$$a_1 = \frac{1}{1},\ a_2 = -\frac{3}{2},\ a_3 = \frac{5}{4},\ a_4 = -\frac{7}{8},\ a_5 = \frac{9}{16},\ a_6 = -\frac{11}{32},\ ...$$

(a) (6 pts) Assuming that the pattern continues, find the formula for the general term $a_n$.

(b) (6 pts) Using your answer from (a), is the sequence $\{a_n\}_n$ convergent? Briefly justify your answer.

**2.** (12 pts) Evaluate each of the following or show it diverges: (6 pts each)

$(a)$ $\displaystyle\int_0^{+\infty} e^{-3x}\,dx$

$(b)$ $\displaystyle\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + ...$

**3.** (10 pts) In each case, answer **True** or **False**. No justification is required for this problem. (2 pts each)

(a) The point of polar coordinates ($r = -2,\ \theta = \frac{9\pi}{4}$) lies in the third quadrant. **True** **False**

(b) Any convergent sequence is bounded. **True** **False**

(c) If $0 \leq a_n \leq \frac{1}{\sqrt{n}}$ for all $n \geq 1$, then $\sum_{k=1}^{\infty} a_n$ is convergent. **True** **False**

(d) If $0 \leq a_n \leq \frac{1}{\sqrt{n}}$ for all $n \geq 1$, then $\lim_{n\to\infty} a_n = 0$. **True** **False**

(e) If $\sum_{k=1}^{\infty} |a_k|$ is convergent, then $\sum_{k=1}^{\infty} a_k$ is convergent. **True** **False**

**4.** (12 pts) Evaluate each of the following or show it diverges: (6 pts each)

$(a)$ $\lim_{k\to+\infty} \left(\frac{1}{k} - \frac{1}{k+2}\right)$

$(b)$ $\sum_{k=1}^{+\infty} \left(\frac{1}{k} - \frac{1}{k+2}\right)$

**5.** (14 pts) (a) (6 pts) Sketch in the $xy$-plane the cardioid $r = 1 - \sin\theta$. Give the polar coordinates of at least four points.

(b) (8 pts) Set up an expression that gives the area inside the circle $r = 1/2$ but outside of the cardioid $r = 1 - \sin\theta$ (you DO NOT have to evaluate the integral(s) in your expression).

**6.** (16 pts) Determine whether each of the following series converges or diverges. Full justification is required.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

**7.** (20 pts) For each of the following series, determine if the series is absolutely convergent (AC), conditionally convergent (CC), or divergent (D). Answer **and carefully** justify your answer. Very little credit will be given just for a guess. Most credit is given for the quality of the justification. (10 pts each)

(a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n \frac{(2n)!}{(n!)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{\sqrt{2n-1}}$

**8.** (12 pts) Choose ONE. If you have time to do both, the second proof may give some bonus towards an earlier lower score.

(a) State and prove the geometric series theorem.

(b) State and prove the $n$th-term test for divergence.