Quiz 1 MAC-2312 Fall 2019 **Name:** Solution Key

To receive credit you MUST SHOW ALL YOUR WORK.

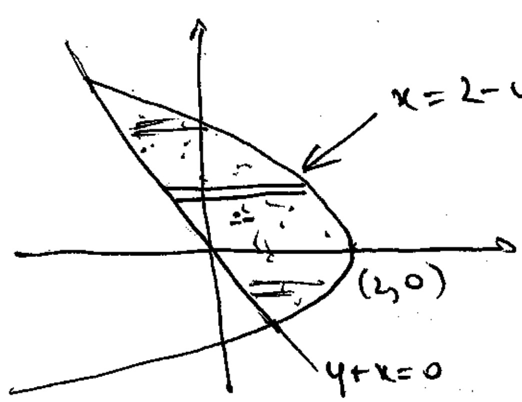
1. (3 pts) Find the average value of the function $f(x) = 1/x$ on the interval $1 \leq x \leq 3$.

We use: f_{ave} when $x \in (a, b)$ $= \dfrac{\int_a^b f(x)\,dx}{b-a}$

so, in our case, $f_{ave} = \dfrac{\int_1^3 \frac{1}{x}\,dx}{3-1} = \frac{1}{2}\left(\ln|x| \Big|_{x=1}^{x=3}\right)$

$f_{ave} = \frac{1}{2}(\ln 3 - \ln 1) = \boxed{\frac{\ln 3}{2}}$

2. (3 pts) Set up an integral that gives the area of the region bounded between the curves $x = 2 - y^2$, $y + x = 0$. You do NOT have to evaluate the integral. Computation is NOT required. Just the set up is required, but a picture of the region should be part of your work.



$A = \int_?^? l_{strip} \cdot Th_{strip}$

As we use horizontal strips (this is the better choice)

$Th_{strip} = dy$

$l_{strip} = x_2 - x_1 = 2 - y^2 - (-y) = 2 - y^2 + y$

Intersection pts:

$\begin{cases} x = 2 - y^2 \\ x = -y \end{cases}$

$2 - y^2 = -y \Rightarrow y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y_1 = -1, \ y_2 = 2$

so $\boxed{A = \int_{y=-1}^{y=2} (2 + y - y^2)\,dy}$

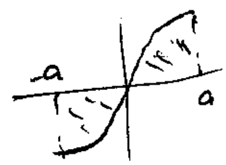
3. (6 pts) Evaluate each integral (3 pts each):

(a) $\int_{-1}^{1} t^3\sqrt{2+t^4}\,dt = *$

Quick way: Observe that $f(t) = t^3 \cdot \sqrt{2+t^4}$ is an odd function (since $f(-t) = -f(t)$)

so its graph is symmetrical with respect the origin.

For any odd function then $\int_{-a}^{a} f(t)\,dt = 0$, using this symmetry



slightly longer way: With subst.

$u = 2 + t^4$

$du = 4t^3\,dt$

$\frac{du}{4} = t^3\,dt$

so $* = \int_{u=3}^{u=3} \sqrt{u}\,\frac{du}{4} = 0$ because the interval is degenerate

(b) $\int_0^{\pi/2} \frac{\cos x}{2+3\sin x}\,dx =$

substitution $w = 2 + 3\sin x$ (when $x=0$ $w = 2+3\sin 0 = 2$; when $x = \frac{\pi}{2}$ $w = 2+3\sin\frac{\pi}{2} = 5$)

$dw = 3\cos x\,dx$

$\frac{1}{3}\,dw = \cos x\,dx$

$$= \int_{w=2}^{w=5} \frac{\frac{1}{3}\,dw}{w} = \frac{1}{3}\ln|w|\Big|_{w=2}^{w=5} = \frac{1}{3}(\ln 5 - \ln 2)$$