

1. Evaluate (a)  $\int \sin^2 x \cos^3 x \, dx$

(b)  $\int \tan^2 x \sec^4 x \, dx$

2. The region bounded between the graph of  $\sin x$  and the  $x$ -axis when  $x \in [0, \pi]$  is rotated around the  $y$ -axis; the solid formed has volume  $V_1$ . Then the same region is rotated around the  $x$ -axis; the solid formed has volume  $V_2$ . Find  $V_1$  and  $V_2$  and observe that  $V_1 = 4V_2$ .

3. (a) Derive a reduction formula for

$$\int \sin^n x \, dx ,$$

where  $n$  is a positive integer. You may check formula (3), with  $m = 0$ , on bottom of page 494 textbook to confirm your result.

(b) Use part (a) to derive a recursion formula for

$$A_n = \int_0^{\pi/2} \sin^n x \, dx .$$

(c) Find  $A_1$  directly, then find  $A_3, A_5$  using the recursion formula. Write a general formula for  $A_n$  when  $n$  is odd.

(d) Find  $A_0$  directly, then find  $A_2, A_4$  using the recursion formula. Write a general formula for  $A_n$  when  $n$  is even.

The general formulas for  $A_n$  are the so-called *Wallis sine formulas*.