Worksheet 10/17/19 - MAC 2312 Group nr. ____ NAMES: Solution Key

1. (2 pts) Write the general form of the partial fraction decomposition for $\frac{2x+5}{(x^3+x^2)(x^2+1)^2}$. You **do not** have to determine the constants involved.

$$\frac{2x+5}{x^2(x+1)(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

2. (8 pts) Use partial fractions to compute the integrals

(a) $\displaystyle\int \frac{x^3}{x^2-4}\,dx$

(b) $\displaystyle\int \frac{1}{x^3+4x}\,dx$

Note: Each integral can be also done with an appropriate trig substitution. You will receive one additional point (for each) if you compute them both ways.

(a)

$$\begin{array}{r|l} & x \leftarrow \text{quotient} \\ x^2-4 & x^3 \\ & -x^3+4x \\ \hline & \quad 4x \leftarrow \text{remainder} \end{array}$$

so $\dfrac{x^3}{x^2-4} = x + \dfrac{4x}{x^2-4}$

$$\frac{4x}{x^2-4} = \frac{4x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$4x = A(x+2) + B(x-2)$$

$x=2 \Rightarrow 8 = 4A \Rightarrow A = 2$

$x=-2 \Rightarrow -8 = -4B \Rightarrow B = 2$

so $$\int \frac{x^3}{x^2-4}\,dx = \int\left(x + \frac{2}{x-2} + \frac{2}{x+2}\right)dx$$

$$= \boxed{\frac{x^2}{2} + 2\ln|x-2| + 2\ln|x+2| + c}$$

(b)

$$\frac{1}{x^3+4x} = \frac{1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$1 = A(x^2+4) + (Bx+C)x$$

$$1 = Ax^2 + 4A + Bx^2 + Cx$$

$$1 = (A+B)x^2 + Cx + 4A$$

so $\begin{cases} A+B=0 \\ C=0 \\ 4A=1 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{4} \\ B = -\frac{1}{4} \\ C = 0 \end{cases}$

Thus

$$\int \frac{1}{x^3+4x}\,dx = \int\left(\frac{\frac{1}{4}}{x} + \frac{-\frac{1}{4}x}{x^2+4}\right)dx$$

$$= \frac{1}{4}\ln|x| - \frac{1}{4}\cdot\frac{1}{2}\ln|x^2+4| + c$$

$$= \boxed{\frac{1}{4}\ln|x| - \frac{1}{8}\ln(x^2+4) + c}$$

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2. (8 pts) Use partial fractions to compute the integrals

(a) $\int \frac{x^3}{x^2-4}\,dx$

(b) $\int \frac{1}{x^3+4x}\,dx = \int \frac{1}{x(x^2+4)}\,dx = *$

Note: Each integral can be also done with an appropriate trig substitution. You will receive one additional point (for each) if you compute them both ways.

(a) with a trig. sub.

let $x = 2\sec\theta$

$x^2-4 = 4\sec^2\theta - 4 = 4(\sec^2\theta - 1) = 4\tan^2\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$$\int \frac{x^3}{x^2-4}\,dx = \int \frac{8\sec^3\theta}{4\tan^2\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$= 4\int \frac{\sec^4\theta}{\tan\theta}\,d\theta = 4\int \frac{\sec^2\theta}{\tan\theta}\cdot\sec^2\theta\,d\theta$$

$$= 4\int \frac{\tan^2\theta+1}{\tan\theta}\cdot\sec^2\theta\,d\theta = 4\int \frac{w^2+1}{w}\,dw$$

$w = \tan\theta$

$dw = \sec^2\theta\,d\theta$

$$= 4\int \left(w + \frac{1}{w}\right)dw = 4\left(\frac{w^2}{2} + \ln|w|\right) + c$$

$$= 2\tan^2\theta + 4\ln|\tan\theta| + c =$$

$\frac{x}{2} = \sec\theta \Rightarrow$  $\Rightarrow \tan\theta = \frac{\sqrt{x^2-4}}{2}$

$$= 2\left(\frac{\sqrt{x^2-4}}{2}\right)^2 + 4\ln\left|\frac{\sqrt{x^2-4}}{2}\right| + c =$$

$$= 2\cdot\frac{(x^2-4)}{4} + 4\left(\ln\left[(x^2-4)^{\frac{1}{2}}\right] - \ln 2\right) + c$$

$$= \frac{x^2-4}{4} + 2\left(\ln|x-2| + \ln|x+2|\right) - 4\ln 2 + c$$ same answer as with partial fractions

(b) with a trig sub.

$x = 2\tan\theta$

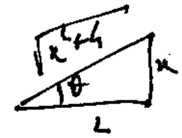
$x^2+4 = 4\tan^2\theta + 4 = 4(\sec^2\theta)$

$dx = 2\sec^2\theta\,d\theta$

$$* = \int \frac{\cancel{2\sec^2\theta}\,d\theta}{\cancel{2}\tan\theta\cdot 4\cancel{\sec^2\theta}} = \frac{1}{4}\int \frac{1}{\tan\theta}\,d\theta =$$

$$= \frac{1}{4}\int \cot\theta\,d\theta =$$

$$= -\frac{1}{4}\ln|\csc\theta| + c =$$

$\frac{x}{2} = \tan\theta \Rightarrow$ 

$\csc\theta = \frac{\sqrt{x^2+4}}{x}$

$$= -\frac{1}{4}\ln\left|\frac{\sqrt{x^2+4}}{x}\right| + c =$$

$$= -\frac{1}{4}\left(\ln(\sqrt{x^2+4}) - \ln|x|\right) + c =$$

$$= \frac{1}{4}\ln|x| - \frac{1}{4}\cdot\frac{1}{2}\ln(x^2+4) + c$$

$$= \frac{1}{4}\ln|x| - \frac{1}{8}\ln(x^2+4) + c$$

same answer as with partial fractions.