**Background Check – Calculus 2** - Spring 2015 **NAME:** Solution Key

**1.** (4 pts) Fill in the exact values:

$\ln(\sqrt{e}) = \ln(e^{\frac{1}{2}}) = \frac{1}{2}$ $\arctan(1) = \frac{\pi}{4}$

$(0.01)^{-1/2} = \left(\frac{1}{100}\right)^{-\frac{1}{2}} = \left(\frac{100}{1}\right)^{\frac{1}{2}} = \sqrt{100} = 10$ $\sin(\frac{7\pi}{6}) = -\frac{1}{2}$

**2.** (6 pts) Circle the correct answer (assume that $x \neq 0$):

(a) The expression $\dfrac{3x^2}{x^4+9x^2}$ is equivalent with:

(*i*) $\dfrac{1}{x^2+3}$ (*ii*) $\dfrac{3}{x^2}+\dfrac{1}{3}$ (*iii*) $\dfrac{1}{x^4+3}$ (*iv*) $\dfrac{3}{x^2+9}$ (*v*) $\dfrac{2}{3x^2}$

(b) The expression $\dfrac{x^2}{\sqrt[3]{x^2}}$ is equivalent with:

(*i*) $\sqrt{x}$ (*ii*) 1 (*iii*) $x\sqrt[3]{x}$ (*iv*) $x^{-1/3}$ (*v*) none of the above

(c) $\displaystyle\lim_{x\to+\infty}\frac{x^2+2}{3x^2+4} =$ (*i*) 1 (*ii*) 3/7 (*iii*) 1/3 (*iv*) 1/2 (*v*) other

**3.** (10 pts) Compute the following limits

(a) $\displaystyle\lim_{x\to 0}\frac{x^2}{1-\cos(3x)} \overset{\frac{0}{0}}{\underset{l'H}{=}} \lim_{x\to 0}\frac{2x}{3\sin(3x)} = \lim_{x\to 0}\frac{\frac{2}{3}\cdot(3x)}{3\cdot\sin(3x)} = \frac{\frac{2}{3}}{3} = \boxed{\frac{2}{9}}$

(b) $\displaystyle\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x \overset{1^\infty}{=} \lim_{x\to+\infty} e^{\ln\left[\left(1+\frac{1}{x}\right)^x\right]}$

$\displaystyle= \lim_{x\to+\infty} e^{x\ln(1+\frac{1}{x})} = e^{\lim_{x\to+\infty}\left[x\ln(1+\frac{1}{x})\right]}$

$\displaystyle\lim_{x\to+\infty} x\cdot\ln\left(1+\frac{1}{x}\right) \overset{\infty\cdot 0}{=} \lim_{x\to+\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$

$\displaystyle\underset{\substack{\uparrow\\ \text{subst.}\\ y=\frac{1}{x}}}{=} \lim_{y\to 0^+}\frac{\ln(1+y)}{y} \overset{\frac{0}{0}}{\underset{l'H}{=}} \lim_{y\to 0}\frac{\frac{1}{1+y}}{1} = 1$

Thus,

$$\boxed{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = e}$$

**4.** (5 pts) Compute $\frac{d}{dx}(\tan^{-1}(e^{3x}))$. Recall that $\tan^{-1}$, also denoted arctan, is the inverse function of tan.

$$\frac{d}{dx}\left(\tan^{-1}(e^{3x})\right) \underset{\substack{\uparrow \\ \text{Chain Rule}}}{=} \frac{1}{1+(e^{3x})^2}\cdot(e^{3x})' = \frac{1}{1+e^{6x}}\cdot 3e^{3x} = \boxed{\frac{3e^{3x}}{1+e^{6x}}}$$

**5.** (5 pts) Compute $g''(x)$ if $g(x) = \sin(x^2)$.

$$g'(x) = (\sin(x^2))' \underset{\text{Rule}}{\overset{\text{Chain}}{=}} \cos(x^2)\cdot 2x = 2x\cos(x^2)$$

$$g''(x) = (2x\cos(x^2))' \underset{\text{Product Rule}}{=} (2x)'\cos(x^2) + 2x(\cos(x^2))' = 2\cos(x^2) - 2x\cdot\sin(x^2)\cdot 2x = \boxed{2\cos(x^2) - 4x^2\sin(x^2)}$$

**6.** (20 pts) Compute the following anti-derivatives (5 pts each)

(a) $\int (2x^3 + \sec^2 x - \frac{1}{3x^2})\, dx =$

$$= \int\left(2x^3 + \sec^2 x - \tfrac{1}{3}x^{-2}\right)dx$$

$$= 2\frac{x^4}{4} + \tan x + \tfrac{1}{3}x^{-1} + c$$

$$= \boxed{\tfrac{1}{2}x^4 + \tan x + \frac{1}{3x} + c}$$

(b) $\int x^2\sqrt{x^3+9}\, dx =$

subst. $w = x^3 + 9$

$dw = 3x^2 dx$

$\frac{1}{3}dw = x^2 dx$

$$= \int \sqrt{w}\,\tfrac{1}{3}dw = \tfrac{1}{3}\int w^{\frac{1}{2}}dw$$

$$= \tfrac{1}{3}\cdot\tfrac{2}{3}w^{\frac{3}{2}} + c = \boxed{\tfrac{2}{9}(x^3+9)^{\frac{3}{2}} + c}$$

(c) $\int \frac{1}{\sqrt{4-x^2}}\, dx = \int \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}dx$

$$= \int \underbrace{\frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}dx}_{dw} =$$

sub. $w = \frac{x}{2}$

$dw = \frac{1}{2}dx$

$$= \int \frac{dw}{\sqrt{1-w^2}} = \arcsin(w) + c$$

$$= \boxed{\arcsin\left(\tfrac{x}{2}\right) + c}$$

(d) $\int \frac{1}{\ln x}\underbrace{\frac{dx}{x}}_{dw} = \int \frac{1}{w}dw =$

sub. $w = \ln x$

$dw = \frac{1}{x}dx$

$$= \ln w + c = \boxed{\ln(\ln x) + c}$$