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Quiz 1 - MAC 2312, Spring 2015

1. (a) (2 pts) Is the sequence $a_n = (-1)^n \frac{n}{2n+1}$ bounded? Answer and briefly justify.

Yes, $\{a_n\}_n$ is bounded. Note that $|a_n| = \frac{n}{2n+1} \leq \frac{1}{2}$, so $-\frac{1}{2} \leq a_n \leq \frac{1}{2}$ for all $n \geq 1$.

(b) (2 pts) Is the sequence $a_n = (-1)^n \frac{n}{2n+1}$ convergent? Answer and briefly justify.

($-1 \leq a_n \leq 1$ for all $n\geq$ also works

No, $\{a_n\}_n$ is not convergent.

The sequence $|a_n| = \frac{n}{2n+1}$ converges to $\frac{1}{2}$, but $a_n = (-1)^n \frac{n}{2n+1}$ diverges

Subsequence $a_{2k} \to \frac{1}{2}$

but $a_{2k+1} \to -\frac{1}{2}$

2. (3 pts) Find the exact value of the sum $1 + 4 + 7 + 10 + 13 + \ldots + 301$.

$$1+4+7+10+\ldots+301 =$$

$$= \sum_{k=0}^{100} (1+3k) = \sum_{k=0}^{100} 1 + 3\sum_{k=0}^{100} k =$$

$$= 101 + 3\sum_{k=1}^{100} k = 101 + 3 \cdot \frac{100 \cdot 101}{2} = 101 \times 151$$

Gauss's trick also works for # 2

$$S = 1+4+7+\ldots+301$$

$$S = 301+298+295+\ldots+1$$

$$2S = \underbrace{302+302+\ldots+302}_{101 \text{ times}}$$

3. (3 pts) Determine if $\sum_{k=2}^{\infty}\left(\frac{1}{k-1} - \frac{1}{k+1}\right)$ converges and if so find its limit.

$$S_n = \sum_{k=2}^{n}\left(\frac{1}{k-1} - \frac{1}{k+1}\right) \underset{\text{cancellation every two}}{\overset{\text{telescopic}}{=}}$$

$$= \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \ldots +$$

$$+ \left(\cancel{\frac{1}{n-2}} - \frac{1}{n}\right) + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right)$$

$$S_n = \frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$ (two terms "survive" at the beginning and two at the end

So $$\sum_{k=2}^{\infty}\left(\frac{1}{k-1} - \frac{1}{k+1}\right) = \lim_{n\to+\infty} S_n = \lim_{n\to+\infty}\left(\frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}\right) = \frac{3}{2}$$

Thus the series converges to $\frac{3}{2}$.