Worksheet week 2   MAC-2312 - Spring 2015   **Name:** _____

1. Consider the sequence:

$$a_1 = \sqrt{3}, \quad a_2 = \sqrt{3 + 2\sqrt{3}}, \quad a_3 = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3}}}, \quad a_4 = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + 2\sqrt{3}}}}, \; ...$$

(a) Find a recursion formula for a_{n+1}.

(b) Use induction to prove that $0 \le a_n \le 3$, for all $n \ge 1$.

(c) Use induction to prove that the sequence $\{a_n\}$ is increasing.

(d) By (b) and (c) it follows that the sequence is convergent (why?). Find its limit.