

Name: \_\_\_\_\_

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Exam 2 - MAC2311 -

Summer B 2019

**Important Rules:**

1. Unless otherwise mentioned, to receive full credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, **NOT** in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should **NOT** be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (16 points) These are True or False questions. Circle your answer and give a brief justification (4 pts each).

(a)  $\ln(A + B) = \ln A + \ln B$ , for any  $A, B$ .                      **True**                      **False**

**Justification:**

(b) If  $y = \sin(5x)$ , then  $y'' = -25y$ .                      **True**                      **False**

**Justification:**

(c) If  $g(x) = x^2 f(x)$ , then  $g'(x) = 2x f'(x)$ .                      **True**                      **False**

**Justification:**

(d) If  $g(x) = f(x^5)$  and  $f'(1) = 2$ , then  $g'(1) = 10$ .                      **True**                      **False**

**Justification:**

2. (35 pts) Compute  $\frac{dy}{dx}$  for each of the following functions. Simplify your answer when possible (7 pts each).

a)  $y = 3x^5 - 2\sqrt{x} + 10^x$

b)  $y = \ln(\sec x)$

c)  $y = e^{3x} \sin(5x)$

d)  $y = \cos(\arcsin(\sqrt{x}))$     **+1 bonus point** if you correctly simplify your answer for this one.

e)  $y = (1 + x^2)^{1/x}$     **Hint:** Logarithmic differentiation

3. (12 pts) (a) (7 pts) Use implicit differentiation to find  $\frac{dy}{dx}$  if  $2x^3 + 3xy = y^3$ .

(b) (5 pts) Use part (a) to find the equation of the tangent line to the curve  $2x^3 + 3xy = y^3$  at the point  $(1, 2)$ .

4. (12 pts) a) (8 pts) Find the local linear approximation of the function  $f(x) = x^{1/4}$  at  $x_0 = 1$ .

b) (4 pts) Use the result of part (a) to approximate  $\sqrt[4]{0.92}$ .

**5.** (12 pts) Recall that the volume  $V$  of a right circular cylinder is given by  $V = \pi r^2 h$ , where  $r$  is the radius of the base and  $h$  is the height. Suppose we have a cylinder for which both dimensions  $r$  and  $h$  vary with time  $t$ .

(a) (6 pts) How are  $dV/dt$ ,  $dr/dt$  and  $dh/dt$  related?

(b) (6 pts) At a certain instant, the height is 8 cm and is increasing at 0.2 cm/s, while the radius is 10 cm and is decreasing at 0.1 cm/s. How fast is the volume changing at that instant? Give units to your answer. Is the volume increasing or decreasing at that instant?

**6.** (12 pts) A fighter-jet is climbing at a  $30^\circ$  angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 250 ft/sec ?

7. (12 pts) Chose ONE of the following proofs. If you have time to do both proofs, the second score may give you some bonus towards a previous problem with a lower score.

(A) Use logarithmic differentiation to prove the power rule formula for derivative.

(B) Find, with proof, the formula for  $(\arctan x)'$ .