Quiz 1 MAC-2311 Summer B 2019 Name: Solution Key

To receive credit you MUST SHOW ALL YOUR WORK.

1. Compute each of the following limits. If the limit does not exist or is infinite, specify so (2.5 pts each).

(a) $\displaystyle\lim_{x\to 3}\frac{x^3-9x}{x^2-4x+3} \overset{\frac{0}{0}}{=}$

$$= \lim_{x\to 3}\frac{x(x^2-9)}{(x-3)(x-1)} = \lim_{x\to 3}\frac{x\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-1)} =$$

$$= \frac{3\cdot 6}{2} = \boxed{9}$$

(b) $\displaystyle\lim_{x\to -\infty}\frac{x^3-9x}{x^2-4x+3} \underset{\text{Junk Rule}}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to-\infty}\frac{x^3}{x^2}$

$$= \lim_{x\to-\infty} x = \boxed{-\infty}$$

(c) $\displaystyle\lim_{x\to 0}\frac{1-\cos(3x)}{x^2} \overset{\frac{0}{0}}{=}$

$$= \lim_{x\to 0}\frac{(1-\cos(3x))(1+\cos(3x))}{x^2(1+\cos(3x))} =$$

$$= \lim_{x\to 0}\frac{\sin^2(3x)}{x^2(1+\cos(3x))} = \lim_{x\to 0}\frac{\frac{\sin^2(3x)}{(3x)^2}\cdot 9\cancel{x^2}}{\cancel{x^2}(1+\cos(3x))}$$

$$= \frac{1\cdot 9}{2} = \boxed{\frac{9}{2}}$$

(d) $\displaystyle\lim_{x\to 2}\frac{2x-7}{x^2-4x+4} = \frac{-3}{0}$

$$= \lim_{x\to 2}\frac{2x-7}{(x-2)^2}$$ ← So the denominator is a positive number both when $x<2$ or $x>2$

$$\frac{-3}{0^+} = \boxed{-\infty}$$

2. (Bonus 2 pts) Does the function $f(x) = \dfrac{x^3-9x}{x^2-4x+3}$ have any vertical or horizontal asymptotes?

Briefly justify. Note that in Pb. 1 (a), (b), you computed some limits of this function.

As in (b), $\displaystyle\lim_{x\to+\infty}\frac{x^3-9x}{x^2-4x+3} = \lim_{x\to+\infty}\frac{x^3}{x^2} = +\infty$

so $f(x)$ has no horizontal asymptotes (as both $\displaystyle\lim_{x\to+\infty} f(x)$ and $\displaystyle\lim_{x\to-\infty} f(x)$ are infinite)

$f(x) = \dfrac{x(x-3)(x+3)}{(x-3)(x-1)}$, so, by (a), $x=3$ is NOT a vertical asymptote ($x=3$ is a removable discontinuity for $f(x)$)

$\boxed{x=1 \text{ is a vertical asymptote}}$

$$\lim_{x\to 1^+}\frac{x\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-1)} = \frac{3}{0^+} = +\infty, \quad \lim_{x\to 1^-}\frac{x(x+3)}{x-1} = \frac{3}{0^-} = -\infty$$