

Name: _____

Panther ID: _____

Exam 1

Calculus II

Fall 2009

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) Circle the correct answer:

(a) A particle is moving along the x -axis, its position at time t being given by the coordinate at that moment, $x(t)$. The total distance traveled by the particle in the time interval $[t_1, t_2]$ is given by

(i) $\frac{x(t_2) - x(t_1)}{t_2 - t_1}$ (ii) $x(t_2) - x(t_1)$ (iii) $\int_{t_1}^{t_2} x(t) dt$ (iv) $\int_{t_1}^{t_2} |x'(t)| dt$ (v) $\int_{t_1}^{t_2} x'(t) dt$

(b) The arc length of $y = \sin x$ from $x = 0$ to $x = \pi$ is given by

(i) $\frac{\sin \pi - \sin 0}{\pi - 0}$ (ii) $\int_0^\pi \sin x dx$ (iii) π (iv) $\int_0^\pi \sqrt{1 + \cos^2 x} dx$ (v) $\int_0^\pi \sqrt{1 + \sin^2 x} dx$

(c) The average value of $y = \sin x$ on the interval $[0, \pi]$ is

(i) 0 (ii) $\frac{1}{2}$ (iii) $\frac{2}{\pi}$ (iv) 2π (v) $\frac{\pi}{2}$

(d) A water-tank, initially full, starts being drained at the moment $t = 0$. Suppose that $r(t)$ (in gals/min) is the rate of the water flow out of the reservoir at the moment t . The equality $\int_0^{30} r(t) dt = 1000$ says that:

- (i) The reservoir initially contains 1000 gallons of water.
- (ii) After the first half an hour there are 1000 gallons of water left in the tank.
- (iii) In the first half an hour 1000 of gallons of water were drained from the tank.
- (iv) At 30 minutes, the water is flowing out of the tank at the rate of 1000 gals/min.

2. (12 pts) Compute the area of the region above the x -axis, bounded by $x = 2 - y^2$ and $x = -y$.

3. (12 pts) The portion of the graph of $y = \ln x$ for $1 \leq x \leq e$ is rotated around the y -axis. Set up an integral that represents the area of the surface obtained. (Do not try to evaluate the integral. It is not required. Picture is.)

4. (24 pts) Compute each integral (6 pts each):

(a) $\int_0^3 |1-x| dx$

(b) $\int_3^6 \sqrt{6x-x^2} dx$

(c) $\int_0^{\ln 3} \frac{e^x}{e^x+4} dx$

(d) $\int_0^2 \frac{x}{\sqrt{1+2x^2}} dx$

5. (12 pts) A cone-shaped reservoir is 18 ft in diameter across the top and 12 ft deep. Assuming that initially the reservoir is filled with a liquid of density ρ lbs/ft³, set up an integral that represents the work necessary to pump out all the liquid through the top of the reservoir. (It is not required to evaluate the integral.)

6. (12 pts) The region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, $x = 1$ is rotated around the line $x = -1$. Find the volume of the solid obtained. Sketch of the solid is required. Computation is also required for this problem.

7. (10 pts) Find an equivalent expression without any integrals and derivatives:

$$\frac{d}{dx} \left(\int_0^{x^2} \left(\frac{d}{dt} \left(\int_0^{t^2} e^{s^2} ds \right) dt \right) \right)$$

8. (12 pts + 6 bonus) Compute the limit

$$\lim_{n \rightarrow \infty} \frac{1 + 5 + 9 + \dots + (4n + 1)}{n^2}$$

Note: You can do this problem either by finding a closed form for the sum in the numerator and then taking the limit, or by recognizing the expression under limit as the Riemann sum of a certain function on the interval $[0, 1]$. Doing the problem correctly both ways will give you a 6 points bonus.