NAME: ______________________________ **Panther ID:** ____________

Exam 1 - MAC 2313 Spring 2007

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (15 pts) Given the vectors $\mathbf{u} = -\mathbf{i} + \mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{j} - \mathbf{k}$, find each of the following:

(a) $\|\mathbf{v} - 2\mathbf{u}\| =$

(b) the angle between the vectors $\mathbf{v}$ and $\mathbf{w}$.

(c) the volume of the parallelipiped determined by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. Interpret the answer.

2. (12 pts) Find the equation of the plane that contains the point $A(-1, 4, -3)$ and the line $x - 2 = t,\ y + 3 = 2t,\ z = -t$.

3. (15 pts) Circle the correct answer:

(a) The surface $y^2 + 2z^2 = 1$ is a(n)

(i) ellipsoid (ii) hyperboloid (iii) elliptic cylinder (iv) elliptic paraboloid

(b) The surface $x + y^2 + 2z^2 = 1$ is a(n)

(i) ellipsoid (ii) hyperboloid (iii) elliptic cylinder (iv) elliptic paraboloid

(c) The area of the parallelogram with adjacent sides the vectors **u** and **v** is given by:

(i) $\mathbf{u} \cdot \mathbf{v}$ (ii) $\|\mathbf{u}\| + \|\mathbf{v}\|$ (iii) $\mathbf{u} \times \mathbf{v}$ (iv) $\|\mathbf{u} \times \mathbf{v}\|$

(d) Let $\mathbf{r}(t)$ be a curve in 3-space (t is an *arbitrary* parametrization). Denote by $\mathbf{N}(t)$, $\mathbf{B}(t)$ the normal, resp. binormal to the curve. The following two vectors are **always** perpendicular:

(i) $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ (ii) $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ (iii) $\mathbf{B}(t)$ and $\mathbf{r}'(t) \times \mathbf{r}''(t)$ (iv) $\mathbf{r}'(t)$ and $\mathbf{N}(t)$

(e) If a particle travels with constant speed along a curve, then

(i) $\mathbf{a} = 0$ (ii) $a_T = 0$ (iii) $a_N = 0$ (iv) displacement is 0

(Notation: $\mathbf{a}$ = acceleration vector; a_T (a_N) = the tangential (normal) component of acceleration.)

4. (12 pts) At the time $t = 0$ an object of mass m is launched from a height s_0 above the ground with an initial velocity vector $\mathbf{v}_0$ which makes an angle α with the horizontal. Denote by v_0 the initial speed of the object ($v_0 = \|\mathbf{v}_0\|$). Starting from Newton's second law, $\mathbf{F} = m\mathbf{a}$, derive the parametric equations of motion.

5. (12 pts) Show that the line of intersection of the planes $x + 2y - z = 2$ and $3x + 2y + 2z = 7$ is parallel to the line $x = 1 - 6t,\ y = 3 + 5t,\ z = 2 + 4t$.

6. (16 pts) Given the helix $\mathbf{r}(t) = 3\cos(2t)\,\mathbf{i} + 3\sin(2t)\,\mathbf{j} + 4t\,\mathbf{k}$, do the following:

(a) (8 pts) Find the parametric equations of the tangent line to the helix when $t = 0$.

(b) (8 pts) Find the arc-length parametrization of the helix with $t = 0$ as the reference point.

7. (6 pts) Show that $\dfrac{d}{dt}(\mathbf{r}(t)\times\mathbf{r}'(t)) = \mathbf{r}(t)\times\mathbf{r}''(t)$

8. (10 pts) (a) Determine the type of the quadric surface $z^2 - x^2 - 2y^2 = 4$ and sketch its graph.

(b) What is the intersection of the surface $z^2 - x^2 - 2y^2 = 4$
with the plane $z = 3$? What about and $z = 1$?

9. (12 pts) Find the curvature $k(t)$ of the plane curve $\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\,\mathbf{j}$.