Sketch of proof of the Euler line Theorem. It is slightly different than what was suggested in class. Fill in the details.

Theorem (Euler): In an arbitrary triangle $\triangle ABC$ the orthocenter H, the centroid G and the circumcenter O are always collinear. Moreover, $|HG| = 2|GO|$.

Sketch of Proof: Let M and N be, respectively, the midpoints of segments $\overline{BC}$ and $\overline{AC}$ and let D and E be the feet of the altitudes from A and B.

Step 1. Show that $\triangle AHB \sim \triangle OMN$, using (AA). Deduce that $|AH| = 2|OM|$.

Let now Q be the point of intersection of the line AM with the line HO. We'll prove that $Q = G$, the centroid. To see this:

Step 2. Use (AA) to show $\triangle AQH \sim \triangle MQO$. Deduce that $|AQ| = 2|QM|$, thus Q trisects the median from A, so it must be the centroid G. From this similarity, you also get $|HG| = 2|GO|$.

For the proof to be completely clean, before step 2, you should also argue that the point Q of intersection of the lines AM and HO exists **and** that Q is between A and M and Q is between H and O. Can you justify these? How will the figure change if the triangle $\triangle ABC$ has an obtuse angle at A? Argue that the proof still works though.