Name: ____________________ **PanthID:** ____________________

Homework 2 MTG 3212 Spring 2010

1. (10 pts) (a) (Midline property of a triangle) Show that in a triangle $\triangle ABC$ if M, N are midpoints of sides AB and AC, then the segment MN is parallel to BC and half as long as BC.

(b) Suppose $ABCD$ is an arbitrary quadrilateral in the plane. Show that the midpoints of AB, BC, CD, DA form a parallelogram.

Note: I would like you to prove both parts of Problem 1 using vectors (it is not difficult), but if you are not able to do it with vectors, any solution is better than none!

Bonus question: (2 pts) Is the property still true if the points A, B, C, D are in 3-space and they are not coplanar?

2. (10 pts) In class we showed how an affine map of the Euclidean space $\mathcal{E}^n$ induces a linear map of the vector space $\mathbb{R}^n$. In this problem you are asked to show the converse.

Suppose $f : \mathbb{R}^n \to \mathbb{R}^n$ is a linear map. Consider also two points O, O' in the Euclidean space $\mathcal{E}^n$. Define $\phi : \mathcal{E}^n \to \mathcal{E}^n$, by $\phi(O) = O'$ and $\phi(P) = O' + f(\overrightarrow{OP})$, where by the sum $A + \overrightarrow{\mathbf{v}}$ of a point with a vector, we understand the translation of the point A by the vector $\mathbf{v}$. Show that ϕ is an affine map.