1. (10 pts) In the complex plane $\mathbb{C}$, consider a circle $\mathcal{C}(z_0, r)$. Let z_1, z_3 be two arbitrary complex number and denote by z_2, z_4, respectively, the inverses of z_1, z_3 with respect to the circle $\mathcal{C}(z_0, r)$. Show that the cross-ratio $(z_1 : z_2; z_3 : z_4)$ is a real number.

Hint: You can do the problem avoiding any computations. We have proved a result describing geometrically when the cross-ratio is a real number. With that, you can turn the problem into a geometry problem which is not hard to solve.

2. (10 pts) Consider a fractional linear transformation

$$L(z) = \frac{az+b}{cz+d}\ , \text{ with } a, b, c, d \in \mathbb{R} \text{ and } ad - bc = 1.$$

Show that

$$\mathrm{Im}(L(z)) = \frac{\mathrm{Im}(z)}{|cz+d|^2}\ , \text{ where } \mathrm{Im}(z) = \tfrac{z-\bar{z}}{2i} \text{ denotes the imaginary part of } z \in \mathbb{C}.$$