

1. For each of the following, find a linear fractional transformation that maps:

(a)  $0, 1, i$  onto  $\frac{1}{2}, \frac{2}{3}, \frac{3+i}{6}$  respectively;

(b)  $\infty, i, 1$  onto  $-1, 0, i$  respectively;

(c)  $-1, \infty, i$  onto  $\infty, i, 1$  respectively;

(d) the real axis onto the unit circle (Is the transformation unique in this case?);

(e) the unit circle  $|z| = 1$  onto the circle  $|z - i| = 1$ . (Is the transformation unique in this case?)

2. Related to Pb. 1, part (i), is there a linear fractional transformation that maps the unit circle  $|z| = 1$  onto the circle  $|z - i| = 1$  leaving the points of intersection of these two circles fixed?

3. Find the fixed points of  $w = \frac{z-1}{z+1}$ . What is the image of the disk  $|z| < 1$ ?

4. Show that the composition of two circle inversions is a linear fractional transformation.