Name: ______________________________ **Panther ID:** _______________

Exam 1 **MAP 2302: Summer B 2018**

1. (18 pts) Label each differential equation with its type - exact, separable, homogeneous (as in section 2.2B), linear, Bernoulli (and not linear), or 'none of the above'. You **DO NOT** have to solve any of them.

(a) $e^y x\, dx + e^x y\, dy = 0$

(b) $\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{x}{y^3}$

(c) $(x^2+4xy)\, dx + (2x^2+6y)\, dy = 0$

(d) $(x^2+3y^2)\, dx + (2x^2+y^2)\, dy = 0$

(e) $\dfrac{dy}{dx} + \dfrac{2y}{x} = \cos(y)$

(f) $\dfrac{dy}{dx} + \dfrac{2y}{x} = 3x$

2. (10 pts) Every solution of the differential equation $y'' - 9y = 0$ is of the form $y(x) = c_1 e^{3x} + c_2 e^{-3x}$ for some real constants c_1, c_2. Use this to solve the initial value problem $y'' - 9y = 0$, $y(0) = 2$, $y'(0) = 0$.

3. (15 pts) Check that the following differential equation is exact and find its general solution (implicit OK).

$$(4x + 3y^2)\,dx + (6xy + 5)\,dy = 0$$

4. (15 pts) Solve the initial value problem: $2xy\;dx + (x^2 + 1)\;dy = 0$, with $y(1) = 3$.

5. (10 pts) Exactly one person in an isolated island population of 10,000 people comes down with a certain disease on a certain day. Suppose the rate at which this disease spreads is proportional to the product of the number of people who have the disease and the people who do not yet have it. Set up an initial value problem that models the spread of this disease. Mention the type of differential equation, but you **do not have to solve it.**

6. (12 pts) These are True/False questions. Answer and give a brief justification (4 pts each).

(a) The equation $y'' + xy' + 4\sin y = 0$ is a linear, 2nd order, ODE. **True** **False**

Justification:

(b) The way to approach the differential equation

$\left(x\tan\frac{y}{x}+y\right)dx - x\,dy = 0$ is to use the substitution $y = vx$. **True** **False**

Justification:

(c) The IVP $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 3$, has a unique solution on an interval $(1-\delta, 1+\delta)$. **True** **False**

Justification:

7. (18 pts) Solve the following Bernoulli differential equation. Check also for any "lost" solution.

$$\frac{dy}{dx} - \frac{2y}{x} = -x^2y^2$$

8. Choose ONE to prove. Note the different point values.

(A) (10 pts) State the formula for $\mu(x)$ used as an integrating factor for $\frac{dy}{dx} + P(x)y = Q(x)$ and then derive (justify) the formula as done in class.

(B) (12 pts) State and prove the theorem showing that a 1st order homogeneous DE (as in section 2.2 B) can be transformed into a separable DE.

(C) (18 pts) Discover a general theorem to determine when a differential equation $M(x, y)dx + N(x, y)dy = 0$ has an integrating factor of the type $\mu(y)$.