Name: ____________________ **Panther ID:** __________

Exam 2 **MAP 2302: Summer B 2018**

1. (15 pts) These are True/False questions. Answer and give a brief justification (5 pts each).

(a) The UC method can be applied to find a particular solution of $y'' + y = e^x \ln x$ **True** **False**

Justification:

(b) If $y_1(x) = e^x$ and $y_2(x) = e^{2x}$ are solutions of a linear, homogeneous 2nd order ODE with constant coefficients $a_2y'' + a_1y' + a_0y = 0$, then $y_3(x) = e^x \cdot e^{2x} = e^{3x}$ is also a solution. **True** **False**

Justification:

(c) The IVP problem $y'' + xy' + x^2y = 0$, $y(1) = 0$, $y'(0) = 0$, has a unique solution defined on $(-\infty, +\infty)$.

True **False**

Justification:

2. (15 pts) Find the general solution of $y^{(4)} + 3y^{(2)} - 4y = 0$.

3. (12 pts) Using the UC method, write the form of a particular solution of $y'' - y = te^t$. You DO NOT have to spend time to find the coefficients.

4. (15 pts) Find the general solution of the Cauchy-Euler ODE for $x > 0$: $x^2y'' + 5xy' + 3y = 0$.

5. (18 pts) A spring is such that a force of 10 newtons would stretch it 5cm. The spring hangs vertically and a 2-kg mass is attached to it. After this 2-kg mass comes to rest in its equilibrium position, it is pulled down 3cm below this position and released at $t = 0$ (with zero initial velocity). The medium offers resistance equal to $4x'$, where x' is the velocity in centimeters per second.

(a) (6 pts) Set up as an IVP problem.

(b) (10 pts) Solve the IVP to find the displacement function $x(t)$.

(c) (2 pts) Is the motion underdamped (or oscillatory damped), critically damped, or overdamped?

6. (15 pts) Use the VP method to find the general solution of the differential equation: $y'' + y = \sec^3 x$.

Hint: You can use the formulas given in Problem 7(B) (see next page).

7. Choose ONE. Note the different point values.

(A) (10 pts) State and prove a theorem on how to get a particular solution of

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = b_1(x) + b_2(x) ,$$

if you know one particular solution $y_1(x)$ of $a_2(x)y'' + a_1(x)y' + a_0(x)y = b_1(x)$

and another particular solution $y_2(x)$ of $a_2(x)y'' + a_1(x)y' + a_0(x)y = b_2(x)$.

(B) (15 pts) Derive the formulas for $c_1'(x)$ and $c_2'(x)$ from the VP method.
That is, show that if y_1, y_2 are linearly independent solutions of $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$, then a particular solution for $a_2(x)y'' + a_1(x)y' + a_0(x)y = b(x)$ is given by

$$y_p(x) = c_1(x)y_1(x) + c_2(x)y_2(x) \ , \quad \text{where}$$

$$c_1'(x) = -\frac{b(x)y_2(x)}{a_2(x)w(x)} \ , \ c_2'(x) = \frac{b(x)y_1(x)}{a_2(x)w(x)} \quad \text{and } w(x) \text{ denotes the Wronskian of } y_1, y_2.$$

(C) (20 pts) Find the general solution of $(x^2 - 1)y'' - 2xy' + 2y = 0$.