

1. Recognize the type for the DE below and solve it:

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$$

2. (a) Check that the DE $y'' + 4y = 0$ has a family of solutions of the form $y(x) = c_1 \sin(2x) + c_2 \cos(2x)$.
For parts (b) and (c), take for granted that this is the **general** solution of the DE (we'll see this in Chapter 4).

(b) Find a solution for the DE in part (a) satisfying the initial value condition $y(0) = 1$, $y'(0) = 4$, or, if this is not possible, explain.

(c) Find a solution for the DE in part (a) satisfying the boundary value condition $y(0) = 1$, $y'(\pi/4) = 4$, or, if this is not possible, explain.

3. (a) Consider the IVP

$$\frac{dy}{dx} = x \cos^2 y, \quad y(0) = \frac{\pi}{2}.$$

Does the Fundamental Theorem for 1st order ODE apply to guarantee the existence and the uniqueness of the solution for the above IVP? Briefly explain.

(b) Now observe that the DE is separable and try to solve the IVP. Can you explain the apparent contradiction between parts (a) and (b)?