Worksheet 7/02 MAP 2302 - Summer B 2018 Group nr. ___ Names: Solution Key

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (Pb. 2 p. 132 text) Given that $y = x + 1$ is a solution of

$$(x+1)^2y'' - 3(x+1)y' + 3y = 0 ,$$

find a linearly independent solution by reducing the order. Write the general solution.

We do the substitution $y = (x+1)\cdot v$

Then $y' = 1\cdot v + (x+1)v' = v + (x+1)v'$

$y'' = v' + 1\cdot v' + (x+1)v'' = 2v' + (x+1)v''$

and substitute in the original equation

$$(x+1)^2\left[2v' + (x+1)v''\right] - 3(x+1)\cdot\left[v + (x+1)v'\right] + 3(x+1)v = 0 \Leftrightarrow$$

$$\Leftrightarrow (x+1)^3v'' + 2(x+1)^2v' - 3(x+1)^2v' \cancel{-3(x+1)v} \cancel{+3(x+1)v} = 0 \Leftrightarrow$$

$$\Leftrightarrow (x+1)^3v'' - (x+1)^2v' = 0, \text{ or dividing by } (x+1)^2$$

$$(x+1)v'' - v' = 0$$

Substituting $w = v'$ get the 1st order linear ODE

$$(x+1)w' = w .$$

This is also separable so $\frac{dw}{w} = \frac{dx}{(x+1)}$

One solution is $\ln w = \ln(x+1)$, or $w = (x+1)$

Then $v' = w = (x+1)$, so take $v = \frac{1}{2}(x+1)^2$ (just one solution is fine)

Substitute back, to get

$y_2 = \frac{1}{2}(x+1)^3$ a second solution of the initial D.E.

As $y_1 = x+1$ and $y_2 = \frac{1}{2}(x+1)^3$ are independent, they form a fundam. set of solutions for the linear DE.

The general solution is $y = c_1(x+1) + c_2\frac{1}{2}(x+1)^3$ or

$$y = c_1(x+1) + c_2(x+1)^3$$

2. (Pb. 2, p. 143 text) Find the general solution for $y'' - 2y' - 3y = 0$.

$$\lambda^2 - 2\lambda - 3 = 0 \iff (\lambda-3)(\lambda+1) = 0$$

$$\lambda_1 = 3, \quad \lambda_2 = -1$$

$$y_1(x) = e^{3x}, \quad y_2(x) = e^{-x} \quad \text{fundam. set of solutions}$$

General solution $\underline{y(x) = c_1 e^{3x} + c_2 e^{-x}}$

3. (Pb. 44, p. 144 text) Solve the IVP $9y'' - 6y' + y = 0$, $y(0) = 3$, $y'(0) = -1$.

$$9\lambda^2 - 6\lambda + 1 = 0 \iff (3\lambda - 1)^2 = 0 \quad \text{so } \lambda_{1,2} = \frac{1}{3}$$

General solution is $y(x) = c_1 e^{\frac{1}{3}x} + c_2 x . e^{\frac{1}{3}x}$

$$3 = y(0) = c_1$$

$$y'(x) = \frac{1}{3} c_1 e^{\frac{x}{3}} + c_2 e^{\frac{x}{3}}\left(\frac{1}{3}x + 1\right)$$

$$-1 = y'(0) = \frac{1}{3} c_1 + c_2$$

so $c_1 = 3$

$c_2 = -1 - \frac{1}{3} c_1 = -2$

Thus, the solution of the IVP is $\boxed{y(x) = 3 e^{\frac{x}{3}} - 2 x e^{\frac{x}{3}}}$

4. Find the general solution for $y''' - 2y'' + 4y' - 8y = 0$.

charact. equ. $\lambda^3 - 2\lambda^2 + 4\lambda - 8 = 0 \iff$

$$\iff \lambda^2(\lambda - 2) + 4(\lambda - 2) = 0 \iff (\lambda-2)(\lambda^2+4) = 0$$

so roots are $\lambda_1 = 2, \quad \lambda_{2,3} = \pm 2i$

General solution is

$$\underline{y(x) = c_1 e^{2x} + c_2 \cos(2x) + c_3 \sin(2x)}$$