Name: Solution Key PanthID: ________________

Quiz 1 MAD 2104 Summer A 2014

This is a take-home quiz due Monday, May 19. Work should be shown for full credit.

1. (10 pts) In each case, prove or disprove. To disprove, it is enough to give a counterexample. (Note that while a Venn diagram helps, it is not a substitute to a proof or a concrete counterexample.)

(*a*) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$, for all sets A, B, C.

Many examples possible, of course!

The statement is false.

Example: Let $A = B = C = \{a\}$

Then $B \setminus C = \emptyset$ so $A \setminus (B \setminus C) = A \setminus \emptyset = A = \{a\}$

But $A \setminus B = \emptyset$ so $(A \setminus B) \setminus C = \emptyset \setminus C = \emptyset$

Thus $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$

(*b*) $(A \cup B) \times C = (A \times C) \cup (B \times C)$, for all sets A, B, C.

This statement is true.

Proof: $(x,y) \in (A \cup B) \times C \overset{\text{def}}{\iff} x \in A \cup B$ and $y \in C \overset{\text{def of } \cup}{\iff} (x \in A$ or $x \in B)$ and $y \in C$

$\overset{\text{logic}}{\iff}$ ($x \in A$ and $y \in C$) or ($x \in B$ and $y \in C$) $\overset{\text{def of product set}}{\iff}$ $(x,y) \in A \times C$ or $(x,y) \in B \times C$

$\iff (x,y) \in (A \times C) \cup (B \times C)$. Thus $(A \cup B) \times C = (A \times C) \cup (B \times C)$

2. (8 pts) Consider the set $A = \{a, b, c, d\}$. In each case, give an example of a relation $\mathcal{R}$ on A satisfying the conditions. Just give the example, no further justification is needed.

(a) $\mathcal{R}$ is reflexive, symmetric, but not transitive.

$R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a), (b,c), (c,b)\}$

R is clearly reflexive and symmetric.

R is not transitive since $(a,b) \in R$, $(b,c) \in R$, but $(a,c) \notin R$.

(b) $\mathcal{R}$ is reflexive, transitive, but not symmetric and not anti-symmetric.

$R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a), (a,c), (b,c)\}$

R is reflexive & transitive

R is not symmetric as $(a,c) \in R$ but $(c,a) \in R$.

R is not anti-symmetric as $(a,b) \in R$, $(b,a) \in R$ but $b \neq a$.

3. (8 pts) Find a concrete formula for a bijective function between the set of all natural numbers and the set of natural numbers congruent to 2 mod 5. Show that the function you found is indeed bijective.

see second page.

Sol. Pb. 3 Quiz #1

Let A be the set of natural number congruent to 2 mod 5.

$A = \{2, 7, 12, 17, 22, \ldots\}$

Define $f : \mathbb{N} \to A$ by

$$f(k) = 5k+2$$

Note that $5k+2 \in A$ since $5k+2 \equiv 2 \pmod 5$

We show that f is one to one and onto.

f one to one:

Suppose $f(k_1) = f(k_2)$. Then $5k_1+2 = 5k_2+2 \Rightarrow$

$\Rightarrow 5k_1 = 5k_2 \Rightarrow k_1 = k_2$.

Thus, f is one to one.

f onto: Suppose $x \in A$. Then $x \equiv 2 \pmod 5 \Rightarrow$

$\Rightarrow 5 \mid (x-2) \Rightarrow x-2 = 5k$ for $k \in \mathbb{Z}$.

$\Rightarrow x = 5k+2$ for some $k \in \mathbb{Z}$.

Since $x \in \mathbb{N}$, $x \geq 0$, so it follows that $k \geq 0$, thus $k \in \mathbb{N}$.

Thus $x = f(k)$ for some $k \in \mathbb{N}$.

We proved that f is onto