1. (8 pts) Write (in words) the negation of each of the following statements:

(a) I don't have a dog and I don't have a cat.

"I have a dog or I have a cat."

(b) If I have a dog then I don't have a cat.

"I have a dog and I have a cat."

(c) There is someone in this class who has a dog, but doesn't have a cat.

"Every person in this class does not have a dog or has a cat."

Also ≡ For every person in this class, if the person has a dog then the person has a cat too.

(d) There is only one person in this class who has a dog and a cat.

There is no one in this class who has a dog and a cat or there are at least two persons in this class who have a dog and a cat.

2. (8 pts) Determine the truth value of each of the following statements if the domain for all variables consists of all integers. Just the answer (True or False) is enough in each case.

(a) $\exists n\, (n^2 = 6)$

False

(b) $\forall n\, (n-1 \leq n)$

True

(c) $\forall m\, \exists n\, (m^2+1 < n)$

True

(d) $\exists m\, \forall n\, (m^2+1 < n)$

False

3. (10 pts) (a) Is the compound proposition $(p \vee q) \vee (p \to q)$ a tautology, a contradiction, or a contingency? Justify. (A contingency is a compound proposition which is neither a tautology nor a contradiction.)

$(p \vee q) \vee (p \to q)$ is a tautology. Can be checked with truth table or justify as follows: If the conditional statement $(p \to q)$ is true then the whole statement is true (since it is an "or" statement. The statement $(p \to q)$ is false only when p is true and q is false, but in this case $(p \vee q)$ is true, so $(p \vee q) \vee (p \to q)$ is true in all cases.

(b) Are the expressions $(p \to q) \wedge (q \to r)$ and $p \to r$ logically equivalent? Justify.

They are not logically equivalent.

Exp: Suppose p true, q false, r true.

Then $(p \to q)$ is false so $(p \to q) \wedge (q \to r)$ is false

But $p \to r$ is true

Full truth table certainly works as well as justification.