Quiz 3 - Take home - Due Monday, June 2, 2014 NAME: Solution Key

To receive credit you MUST SHOW ALL YOUR WORK.

1. (13 pts) (a) Assuming the pattern continues, find the next two terms of the sequence and give a formula for the general term a_n.

$$a_1 = 3,\ a_2 = 7,\ a_3 = 11,\ a_4 = 15,\ a_5 = 19,\ a_6 = 23,\ \ldots,\ a_n = 4n-1,\ \ldots$$

(b) For the sequence in part (a), find a simple formula for $\sum_{k=1}^{n} a_k$

$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n}(4k-1) \underset{\text{sum properties}}{=} 4\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = 4\cdot\frac{n(n+1)}{2} - n = \boxed{2n^2+n}$$

You may also use that for an arithmetic sum

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \ldots + a_{n-1} + a_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(3 + 4n - 1) = 2n^2 + n$$

but it would be good if you also understand why the formula holds. (Think of Gauss idea)

2. (13 pts) Same exercise as **1.** with both parts (a) and (b), this time for the sequence

$$a_1 = \frac{1}{3},\ a_2 = \frac{1}{15},\ a_3 = \frac{1}{35},\ a_4 = \frac{1}{63},\ a_5 = \frac{1}{9\cdot 11},\ a_6 = \frac{1}{11\cdot 13},\ \ldots,\ a_n = \frac{1}{(2n-1)(2n+1)} \ldots$$

$a_1 = \frac{1}{1\cdot 3}$ $a_2 = \frac{1}{3\cdot 5}$ $a_3 = \frac{1}{5\cdot 7}$ $a_4 = \frac{1}{7\cdot 9}$

also $a_n = \frac{1}{4n^2-1}$

Hint for part (b). Realize that the sum is telescopic. See the example done in class.

For (b)
$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)}$$

With a bit of guessing and adjusting your guess, realize that
$$\frac{1}{(2k-1)(2k+1)} = \frac{1}{2}\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right)$$

Thus
$$\sum_{k=1}^{n} a_k = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right) =$$

$$= \frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \cancel{\ldots} + \left(\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right)\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{2n+1}\right] = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}n}{2n+1} = \boxed{\frac{n}{2n+1}}$$