

To receive credit you MUST SHOW ALL YOUR WORK.

1. (10 pts) Use mathematical induction to prove that for every positive integer n

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + n \cdot 2^n = (n-1)2^{n+1} + 2$$

2. (18 pts) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent and 7-cent stamps.

(a) (4 pts) Determine the truth value of $P(n)$ when $3 \leq n \leq 20$.

(b) (4 pts) Based on part (a), formulate a conjecture of the type: for any $n \geq n_0$ the statement $P(n)$ is true. You should determine the value of n_0 as it emerges from part (a).

(c) (10 pts) Use Mathematical Induction to prove your statement from (b).