Quiz 5 − take home   MAD 2104   Summer A 2011

Due Monday, June 13. For full credit, you must show all your work.

1. (10 pts) Consider the matrix

$$A = \begin{pmatrix} 1 & c \\ 0 & d \end{pmatrix}, \qquad \text{where } c \text{ and } d \text{ are given constants.}$$

Find a formula for A^n, where n is a positive integer, and then use induction to prove your formula.

2. (10 pts) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of 2, that is, as a sum of a subset of the integers $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and so on. For example, for $n = 23$, $23 = 2^4 + 2^2 + 2^1 + 2^0$.

Note and Hint: This is the binary expression of a given number n. One way to establish the inductive step is the following: given $n + 1$, it must fall between two successive powers of 2 (why?). That is, $2^k \leq n + 1 < 2^{k+1}$ for some k. Then consider $n + 1 - 2^k$ and apply the inductive assumption.