Name: ____________________ **Panther ID:** ____________________

Exam 2 MAA 3200 Fall 2009

1. (14 pts) Give a proof of, or a counterexample to, the following statement:
For any three sets A, B, C

$$A \times (B - C) = (A \times B) - (A \times C)$$

2. (14 pts) Let $a_1 = 1$, $a_2 = 6$ and for each natural number $n \geq 2$ let $a_{n+1} = 6a_n - 5a_{n-1}$. Show, by induction, that

$$a_n = \frac{5^n - 1}{4}, \text{ for all } n \geq 1.$$

3. (16 pts) (a) (10 pts) Let p be a prime and let $a \in \mathbb{N}$. Then $a^2 \equiv 1 \pmod{p}$ if and only if $a \equiv 1 \pmod{p}$ or $a \equiv -1 \pmod{p}$.

(b) (6 pts) Find all solutions (mod 8) of the equation $a^2 \equiv 1 \pmod{8}$. This shows you that the statement in (a) may fail if p is not prime.

4. (14 pts) Let b be a nonzero integer and let a, q, r be integers such that $a = qb + r$. Prove that $\gcd(a, b) = \gcd(b, r)$.

5. (14 pts) Our band of pirates, with 15 people now, finds another bag of gold coins. They try to divide the treasure, but one coin remains. As usual, a fight follows and one pirate is killed. They try again to divide the coins among those left, but again one coin remains. Another fight follows and one more pirate is killed. With 13 pirates remaining, they are finally able to divide the treasure evenly. How many coins were in the bag?

6. (20 pts) Let n be an integer and denote $d = \gcd(11n + 2, 6n + 5)$.

(a) (12 pts) Show that either $d = 1$ or $d = 43$.

(b) (8 pts) Find all values of n such that $\gcd(11n + 2, 6n + 5) = 43$.

7. (18 pts) On the set of integers $\mathbb{Z}$ consider the relations R and S given by:

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \ : \ a + b \text{ is even } \} \qquad S = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \ : \ a + b \text{ is odd } \}.$$

(a) (12 pts) Determine if any of the relations R, S is an equivalence relation. Your answer needs complete justification.

(b) (6 pts) For the relation(s) that you determined in part (a) as being equivalence relation, write the partition induced on $\mathbb{Z}$.