Quiz 1 - Linear Algebra Spring 2006 **NAME:** Answer Key

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (a) (4 pts) Define each of the following:

over-determined (linear) system

A system with more equations than unknowns.

inconsistent (linear) system

A system with no solution.

(b) (6 pts) Give an example of an over-determined linear system which has exactly one solution. (Show that your system has indeed just one solution.)

There are, of course, many correct examples. Here is one:

$$\begin{cases} x_1 & +x_2 & = 1 \\ x_1 & -x_2 & = 1 \\ 2x_1 & -2x_2 & = 2 \end{cases}$$

It's over-determined as there are 3 equations and only 2 variables, but the only solution of the system is $(x_1 = 1, x_2 = 0)$. (Observe that the 3equation is just the same as the second, so one can just solve the system formed by the first two equations.)

2. (10 pts) Solve the following system by Gaussian elimination, or by Gauss-Jordan elimination:

$$\begin{cases} x_1 & +3x_2 & +x_3 & +x_4 & = 3 \\ 2x_1 & -2x_2 & +x_3 & +2x_4 & = 8 \\ x_1 & -5x_2 & & +x_4 & = 5 \end{cases}$$

Carefully doing the Gauss-Jordan elimination, one gets the augmented matrix in reduced row-echelon form:

$$\begin{pmatrix} 1 & 0 & -5/8 & 1 & 15/4 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Since we have only two pivots corresponding to x_1 and x_2, it means that x_3 and x_4 are free variables, so the system will have infinitely many solutions. These solutions are of the form:

$$\begin{cases} x_1 & = 5/8\alpha - \beta + 15/4 \\ x_2 & = -1/8\alpha - 1/4 \\ x_3 & = \alpha \\ x_4 & = \beta \end{cases}$$

Note: Certainly you could do this problem, by obtaining only the row-echelon form of the matrix. But, for complete credit, you had to describe the general form of the solutions (just the matrix in RE or RRE form is not enough).