Quiz 1 - Linear Algebra Summer A 2010 **NAME:** ____________________________

This is a take-home quiz. It is due on Monday, May 10. Late submissions will incur penalties. Two of these four problems will be graded for credit. You have to show all your work for full credit.

1. (10 pts) Pb. 9, page 27 textbook.

2. (10 pts) Solve the following system by reducing the augmented matrix of the system to reduced row-echelon form. Is the system consistent? Does it have a unique solution?

$$\begin{cases} x_1 & +2x_2 & +4x_3 & +x_4 & = 4 \\ 2x_1 & +3x_2 & -2x_3 & -3x_4 & = 1 \\ x_1 & +x_2 & & -x_4 & = 0 \\ 2x_1 & +4x_2 & +2x_3 & -x_4 & = 5 \end{cases}$$

3. (10 pts) The goal of this exercise is to deduce the formula for the inverse of a 2x2 matrix.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. We are looking for a matrix $B = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$, so that $AB = BA = I$.

Write the 4x4 linear system in x, y, z, w corresponding to $AB = I$, and observe that the system can be broken in two 2x2 systems with 2 unknowns. Each system will be consistent precisely when $ad - bc \neq 0$, so the matrix A has an inverse if and only if $ad - bc \neq 0$. Solving for x, y, z, w, you should obtain that

$$B = A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

4. (10 pts) (a) In class there was the following question: "Suppose A, B are square matrices. Is it always true that the sum of **all** entries of AB is the same as the sum of all entries of BA?" Show that the answer is "No" by finding a concrete example of 2x2 matrices.

(b) Now consider the following question: "Suppose A, B are square matrices. Is it always true that the sum of the **diagonal** entries of AB is the same as the sum of the diagonal entries of BA?" Investigate several examples with 2x2 matrices to form an idea if the question has an affirmative or a negative answer. Can you prove your answer?

Note: If $C = (c_{ij})$ is an $n \times n$ matrix, then the sum of the diagonal elements of C is $c_{11} + c_{22} + ... + c_{nn}$.