

Due Tuesday, May 31. To receive credit you MUST SHOW ALL YOUR WORK.

1. (12 pts) In P_4 , the set of polynomials of degree less than 4, consider the subset S of polynomials $p(x)$ that satisfy $p(1) = 0$ and $p'(1) = 0$ (p' denotes the derivative of p).

(a) (6 pts) Show that S is a subspace of P_4 .

(b) (6 pts) Find a basis for S .

2. (8 pts) Use a Wronskian to show that for any $n \geq 1$, the functions $p_1(x) = 1$, $p_2(x) = x$, ..., $p_n(x) = x^{n-1}$ are linearly independent in $C(-\infty, \infty)$. From this and Theorem 3.4.1, argue that $C(-\infty, \infty)$ cannot have a finite dimensional basis.