Take home Quiz 6 - Linear Algebra Summer 2011 **NAME:**

Due Monday, June 20. To receive credit you MUST SHOW ALL YOUR WORK.

1. (5 pts) Prove that if $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbf{R}^n$, then

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2 \ .$$

Notes: This is a vector proof of the fact that in any parallelogram, the sum of the squares of the sides equals the sum of the squares of the diagonals. The above property also has a theoretical importance, as it characterizes normed spaces $(V, \|\cdot\|)$ for which the norm is induced by an inner product.

2. (5 pts) Show that if A, B are $n \times n$ orthogonal matrices, then so is AB.

3. (10 pts) Suppose that m points $(x_1, y_1, z_1), (x_2, y_2, z_2), ..., (x_m, y_m, z_m)$ are given in $\mathbf{R}^3$. Your task is to find the plane of the form $z = a + bx + cy$ that represents the best least squares approximation for the given points.

(a) Set up the 3×3 system whose solution will give you the corresponding $(a, b, c)^T$.

(b) Find the best plane for the points $(0,0,0), (0,1,0), (1,0,1), (1,1,3), (1,-1,2)$. Feel free to use MATLAB for part (b).