Homework - Due Tuesday, Nov. 16

The *Möbius function*, $\mu(n)$, is defined by:

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1; \\ (-1)^r & \text{if } n = p_1 p_2 ... p_r, \text{ where } p_i \text{ are distinct primes}; \\ 0 & \text{otherwise.} \end{cases}$$

1. (10 pts) Show that the Möbius function $\mu(n)$ is multiplicative.

2. (10 pts) Show that the function $F(n)$, obtained from the Möbius function by $F(n) = \sum_{d|n} \mu(d)$, satisfies

$$F(n) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1. \end{cases}$$

(Feel free to use the result of the exercise 19.2 (b) from the textbook.)