Name: ______________________________ **Panther ID:** ______________

Final Exam **MAP 2302: Summer B 2019**

1. (14 pts) Answer True or False. No justification is necessary (unless the question looks ambiguous). (2 pts each)

(a) $y = 3e^{2x}$ is a solution for $y' = 2y$. **True** **False**

(b) The general solution for the equation $y'' - 9y = 0$ is $y = c_1 e^{3x} + c_2 e^{-3x}$, with c_1, c_2 constants.

True **False**

(c) The UC method can be applied to find a particular solution of $y'' - 9y = x^2 e^{2x} \sin x$. **True** **False**

(d) The function e^{x^2} has a Laplace transform. **True** **False**

(e) Given that $y = e^x$ is a solution of $(x^2 + x)y'' - (x^2 - 2)y' - (x + 2)y = 0$, a second linearly independent solution can be found using the substitution $y = e^x v$.

True **False**

(f) The functions $\{\sin(2x), \cos(2x)\}$ are linearly dependent. **True** **False**

(g) The IVP problem $y' = xy^2$, $y(1) = 0$, has unique solution the trivial solution $y(x) = 0$. **True** **False**

2. (15 pts) Short answers:

(a) (3 pts) Suppose a simple harmonic motion (from Ch 5.2) has the formula $x(t) = 2\sin(t) + 3\cos(t)$. What is the amplitude of the motion?

(b) (3 pts) Give the standard form of a Bernoulli DE, from Ch. 2.3.B.

(c) (3 pts) Give the formula for an integrating factor μ, for a linear DE $y' + P(x)y = Q(x)$.

(d) (6 pts) Find the singular points of the DE $(x^3 + x^2)y'' + y' + xy = 0$, and state whether they are regular singular or irregular singular points.

3. (15 pts) Find the general solution (implicit form OK) for the first order DE

$$(x^2 + y^2)\,dx + (2xy + y^2)\,dy = 0\ .$$

4. (15 pts) Find the general solution for $y'' + 4y = 16e^{-2x}$. UC method is suggested, although other ways are possible (and acceptable too).

5. (15 pts) Find a series solution in powers of x for the I.V.P. $y'' + xy' - 2y = 0$, $y(0) = 0$, $y'(0) = 1$. OK to list just the first three non-zero terms, but you should also list the recursive relation.

6. (15 pts) Use a Laplace transform to solve this IVP, where δ is the usual Dirac delta function:

$$y' - 2y = \delta(t-3), \quad y(0) = 1 \ .$$

Simplify completely, writing the solution in piecewise form if necessary. Do not worry if $y(t)$ is not continuous.

7. Choose ONE proof, but you could do TWO for possible bonus. Note the different values.

(A) (12 pts) Show that $L(e^{at}) = \frac{1}{s-a}$, for $s > a$.

(B) (18 pts) Compute the convolution of $\sin(bt)$ with itself and explain how the result is linked with the first formula 8 in the table prepared by Christian. Hint: you may need the identity $\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

(C) (18 pts) Justify (as done in class or in the textbook) the formula $L\{\delta(t-t_0)\} = e^{-t_0 \cdot s}$