

1. Consider the 2nd order, linear homogeneous DE

$$(x - 1)y'' - xy' + y = 0.$$

(a) Check that  $y_1(x) = x$  is a solution for the DE.

(b) Use the recipe from Theorem 4.6 (or 4.7) from section 4.1 C to reduce the order and find a second solution  $y_2(x)$  for the DE. The recipe is to do the substitution  $y = y_1(x) \cdot v$  (in this case,  $y = xv$ ), followed by another substitution  $w = v'$ , to get a 1st order linear ODE.

(c) Write the general solution of the DE.

(d) Find the solution of the DE that also satisfies the initial condition  $y(0) = 3$ .

2. Find the general solution of the DE

$$y''' - 5y'' + 7y' - 3y = 0$$

3. Given that

$$m^4 + 6m^3 + 11m^2 + 6m + 1 = (m^2 + 3m + 1)^2$$

find the general solution of the DE

$$y^{(4)} + 6y^{(3)} + 11y^{(2)} + 6y' + y = 0$$