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Quiz 0 MAP 2302 - Summer B 2019

To receive credit you MUST SHOW ALL YOUR WORK.

1. (4 pts) Compute $\frac{dy}{dx}$ in each case:

(a) $y = x^2e^{3x}$ Product & Chain Rule

$\frac{dy}{dx} = 2x\cdot e^{3x} + x^2\cdot e^{3x}\cdot 3$

or $\frac{dy}{dx} = (2x+3x^2)e^{3x}$

(b) $y = \ln(\sin(\sqrt{x}))$ Chain Rule (twice)

$\frac{dy}{dx} = \frac{1}{\sin(\sqrt{x})}\cdot\cos(\sqrt{x})\cdot\frac{1}{2}x^{-\frac{1}{2}}$

or $\frac{dy}{dx} = \frac{\cot(\sqrt{x})}{2\sqrt{x}}$

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2. (4 pts) Compute each anti-derivative:

(a) $\int \frac{\cos x}{2+\sin x}\,dx$

sub. $u = 2+\sin x$

$du = \cos x\,dx$

$\int \frac{du}{u} = \ln|u| + c$

$= \ln|2+\sin x| + c$

$= \ln(2+\sin x) + c$

(as $2+\sin x > 0$ for any x)

(b) $\int \frac{1-x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx$

$= \arctan x - \frac{1}{2}\ln(1+x^2) + c$

(I used "guess & adjust guess" method for the second integral.)

If you prefer, the sub $u = 1+x^2$, $du = 2x\,dx$ so $\frac{1}{2}du = x\,dx$, will certainly get you to the same answer.

3. (3 pts) Newton's Law of Cooling states that the rate of change of the temperature of a cooling body is proportional to the difference between the temperature of the body and the constant temperature of the surrounding medium. A potato that has been baking at 450°F is taken out of the oven and is left to cool down in a room with (constant) temperature of 65°F. Let $y(t)$ be the temperature of the potato t minutes after it was taken out of the oven. Set up a differential equation for $y(t)$ according to Newton's Law of Cooling. You do not have to solve the equation.

The differential equation is

$\frac{dy}{dt} = -k(y-65)$, with k a positive constant.

But the complete story is more suitable for the initial value problem (I.V.P.)

$$\begin{cases} \frac{dy}{dt} = -k(y-65) \\ y(0) = 450 \end{cases}$$

, as you also know the initial temperature of the potato as is taken out of the oven.