Name: ____________________ **Panther ID:** ____________

Take home part of the Final Exam MAT 3501 Fall 2019

This is the take home part of the final exam, due Tuesday, Dec. 4. You are encouraged to collaborate, ask questions, but each of you should understand the solutions that you are handing in. You should also acknowledge collaborations or outside help and also reference any resources that you may have used.

1. Consider the triangle with sides $a = 13, b = 13, c = 6$. Find:

(a) The area of the triangle.

(b) The angles of the triangles (ok to leave them as inverse trig functions).

(c) The inradius r and the circumradius R of the triangle.

(d) The given triangle is a particular example of a *Heronian triangle*, i.e. a triangle whose sides a, b, c are integers and the area is also an integer. Is the following statement true or false: "For any Heronian triangle, the inradius, r, and circumradius, R are rational numbers"? Justify your answer.

(e) The triangle given in (a) is actually an isosceles Heronian triangle. Show that for any isosceles Heronian triangle with $a = b$, the third side, c, is an even integer and that the altitude to that side is an integer. As a consequence of this, you obtain that any isosceles Heronian triangles is formed by joining two congruent Pythagorean triangles (these are right-angle triangles with sides of integer lengths) along a common side. For example, the triangle in example (a) is obtained from joining two Pythagorean triangles $(5, 12, 13)$ along the side of length 12. (Note also that any Pythagorean triangle is automatically Heronian.)

2. Problem 3, page 371 textbook, with the following additional question for each part: after writing the recursive formula, find a formula for the general term of the sequence in each case.

Note: If you are not familiar with the topic, you should read the section 8.2. In particular, you should understand Theorems 8.6, 8.7 and finding the general term as in formula (8.12) for a recursive formula as in example 8.2.

3. For this both parts, it is useful to first read section 10.9 in the textbook.

(a) Problem 2, page 512, textbook. As in Example 10.31 on page 508, you could give a calculus proof or a geometric proof (or both). The geometric proof is much more elegant and shorter.

(b) Suppose P is a fixed point in the interior of a given angle. Find point A and B on each of the sides of the angle so that the perimeter of the triangle $\triangle PAB$ is minimum.

4. The *Koch Snow-flake*, which we'll call K, is one of the most famous and early examples of *fractals*. It is generated recursively as follows. Start with K_1, an equilateral triangle with sides of length one. For each of the sides of K_1 do the following steps:

(1) divide the line segment into three segments of equal length;
(2) draw an equilateral triangle that has the middle segment from step 1 as its base and points outward;
(3) remove the line segment that is the base of the triangle from step 2.

After doing these steps on all sides of K_1, one obtains the second iteration K_2. The process is continued on all sides of K_2 and one obtains the third iteration, K_3, and so on. The Koch snowflake is the curve obtained as this process is done infinitely many times, $K = \lim_{n\to+\infty} K_n$.

Show that the Koch snow flake K has an infinite length, but it bounds a finite area inside (which you should find!).

Note: In your text, on page 388, you have the description of the *Koch curve*. It is obtained with the same procedure, but starting with just one unit length segment rather than a triangle. With a bit more advanced analysis, one can show that the Koch curve gives an example of a function which is everywhere continuous on the interval $[0, 1]$, but nowhere differentiable!

5. The Diagonal Intruder Exploration: from the beautiful book *Exploratory Problems in Mathematics*, by Frederick W. Stevenson, ISBN 0-87353-338-0.

Answer Questions 1 and 2, and, obviously, try to generalize. See attachment.