Name: ____________________ **Panther ID:** __________

Exam 1 MAT 3501 Fall 2019

1. (15 pts) For each of the following, answer if the statement is True or False. Then give a brief justification.

(a) The product of any two rational numbers is rational. **True** **False**

Justification:

(b) $\log_2 3$ is irrational. **True** **False**

Justification:

(c) If p is prime and $p \geq 3$ then $(p+3) \mid p!$. **True** **False**

Justification:

2. (15 pts) (a) Find the prime factorization for the number $N = 49725$.

(b) If $N = 49275$ is the product of the ages of a group of teenagers, how many teenagers are there and what are their ages?

3. (15 pts) Prove (by induction, or otherwise) that for all $n \geq 1$

$$1^3 + 2^3 + 3^3 + \ldots + (n-1)^3 + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

4. (20 pts) A number (or word, or even phrase) is called a *palindrome* if it reads the same forward or backward.

(a) How many palindromes with exactly five digits are there in base 10?

(b) Show that if N is a palindrome in base 10 with an even number of digits, then N is divisible by 11.

(c) Discover and prove a similar statement as in (b) for palindromes in bases other than 10.

(d) Find, if possible, a palindrome N in base 10, so that, written in base 2, it is also a palindrome with 6 digits.

5. (15 pts) (a) Show by example that if $ab \equiv ac \pmod{n}$, then it is NOT necessarily true that $b \equiv c \pmod{n}$. Thus, while we can add, subtract and multiply with mods, division requires care.

(b) Show that if $ab \equiv ac \pmod{n}$ and if a and n are relatively prime, THEN it follows that $b \equiv c \pmod{n}$.

6. (15 pts) You have an unlimited supply of 5 cent stamps and 7 cent stamps.

(a) Describe all possible ways in which you can make 101 cents worth of postage with 5 cent and 7 cent stamps.

(b) Find the smallest positive integer n_0 with the property that for any $n \geq n_0$, a postage of n cents can be realized with 5 cent and 7 cent stamps.

7. (15 pts) Choose ONE of the following proofs. If you do both, the second score may give some bonus towards a previous problem with a lower score.

(A) Show that if a, b are positive integers, then there exist integers m, n so that $ma + nb = \gcd(a, b)$.

(B) Show that there are infinitely many primes.