Name: ______________________ **Panther ID:** ______________________

Worksheet 10/29/2019 MAT 3501 Fall 2019

1. (a) Show directly, using the definition, that $\sqrt[3]{2} \cdot \sqrt{7}$ is algebraic.

(b) Show directly, using the definition, that $\sqrt[3]{2} + \sqrt{7}$ is algebraic.

(c) Show directly, using the definition, that $\sqrt[3]{2 + \sqrt{7}}$ is algebraic.

(d) Show that if a is an algebraic number, then $-a$ is also algebraic.

(e) Show that if a is an algebraic number, $a \neq 0$, then $1/a$ is also algebraic.

(f) Let a be a real number. Show that if there exists $p(x) \in \mathbf{Z}[x]$, a polynomial with integer coefficients so that $p(a)$ is algebraic, then a is an algebraic number.

Note: Generalizing parts (a) and (b), it is true that the set of algebraic numbers $\mathcal{A}$ is closed under addition and multiplication (i.e. sum of algebraic numbers is algebraic and the same for product). You could think about a proof of this fact, but I don't know an elementary proof for either. This, together with parts (d) and (e), will show that $(\mathcal{A}, +, \cdot)$ is a *field* (you can look up the definition).

2. Use the Theorem of Gelfond (Theorem 4.4.18 in handout) to prove Theorem 4.4.20 also in the handout.

Note: You are given already the proof in the handout, so you could just understand it and rewrite it, but you should also do the exercise that was left to you.