Worksheet - 8/29 MAT 3501 – Fall 2019 **Names:** ____________________

1. (Mostly the same as Pb. 3 from previous worksheet) The literature teacher decided to find out who out of 40 students had read the books A, B, C over the summer break. The results were the following: 25 students had read the book A, 22 the book B and 22 the book C; 33 students had read the book A or B, 32 the book A or C, and 31 had read the book B or C; 10 students had read all the three books.

(a) How many students had read at least one of the books?

(b) How many students had read exactly one book (of the three)?

(c) How many students had read none of the books?

2. If A is a finite set (i.e. a set with finitely many elements), denote by $|A|$ the number of elements of A ($|A|$ is also called the *cardinality* of the set A). Suppose that A, B, C are finite sets (with possibly non-empty intersections).
(a) Find a formula for $|A \cup B \cup C|$ in terms of the cardinalities of A, B, C and their intersections.

Note: This is directly related to Problem 1 (a) and the formula you discovered is the so called Principle of Inclusion and Exclusion (for the case of three sets).

(b) In one or two sentences, can you explain the formula as you would explain it for your students (and also justify the name of the principle)?

(c) Can you generalize to obtain the Principle of Inclusion and Exclusion for n finite sets?

3. (Similar to Pb. 4 from the previous worksheet)
(a) Suppose n lines are drawn in the plane so that no two lines are parallel and no three lines are concurrent. Find a formula, in terms of n, for the number of regions determined in the plane.

(b) A region in the plane is called *bounded* if it can be included in a big enough disk (of finite radius) and it is called *unbounded* otherwise. How many of the regions in part (a) are bounded, how many of them are unbounded?

Hint: The obvious hint for such problems is to start with small values of n and investigate for an eventual pattern.