Name: ______________________ **Panther ID:** ____________

Worksheet - Sep. 17 MAT 3501 Fall 2019

1. Use the Euclidean algorithm to find gcd(360, 132). (You can confirm your result by listing the divisors of each and then finding the common divisors.)

(b) Find lcm(360, 132) in two ways: first using the prime factorization of the numbers, then using the stated theorem about the relation between gcd and lcm.

2. Find $gcd(a, b)$ and $lcm(a, b)$ if $a = 2^3 \times 3 \times 5 \times 7$, $b = 2^2 \times 5^3 \times 11$. Explain, in words, how you generalize this to find $gcd(a, b)$ and $lcm(a, b)$ from the prime factorization of the numbers a, b. Also explain why the rules you discovered are consistent with the stated theorem that

$$lcm(a, b) \cdot gcd(a, b) = ab \text{ , for any positive integers } a, b.$$

3. Prove that any two successive Fibonacci numbers F_n, F_{n+1}, $n \geq 2$ are relatively prime. Recall that the Fibonacci sequence is defined recursively by $F_{n+1} = F_n + F_{n-1}$, for $n \geq 1$ and $F_0 = 1, F_1 = 1$.

4. Prove that if $ad - bc = 1$ then the fraction $\frac{a+b}{c+d}$ is irreducible. (Assume that a, b, c, d are all positive integers.)

5. With this exercise we'll prove (without using prime factorization) that

$$lcm(a, b) \cdot gcd(a, b) = ab \text{ .}$$

Fill in the following sketch of proof: Let $D = gcd(a, b)$. Then $a = a_1 \cdot D$, $b = b_1 \cdot D$, for some integers a_1, b_1.

(a) Argue that $gcd(a_1, b_1) = 1$ (that is, a_1 and b_1 must be relatively prime).

(b) Next let $m = \frac{ab}{gcd(a,b)} = a_1 b_1 D$. Note that m is a common multiple of a and b (why?).
It remains to show that M is the *lowest* common multiple.

(c) Let M another common multiple of a and b. We'll show that $m|M$, so $m \leq M$. To get this, write $M = ka = lb$, for some integers k, l, or $M = ka_1 D = lb_1 D$. On the other hand, by part (a), there are integers x, y so that $a_1 x + b_1 y = 1$. Multiply this relation by M and show that both terms in the left side are multiples of m.

6. (a) How many divisors does the number $N = 6000$ have? (It may be easier to think of part (b) first!)

(b) If the prime factorization (using distinct primes) of a number N is $N = p_1^{k_1} p_2^{k_2} \ldots p_s^{k_s}$, how many divisors does the number N have? (Hint: If $d|N$, think of the prime factorization of d.)